

PROGRESSIVE FAILURE OF LAMINATED COMPOSITE CYLINDRICAL SHELLS

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Prepared By

Jayashree Sengupta

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Under the Guidance of

PROF. DR. DIPANKAR CHAKRAVORTY

**DEPARTMENT OF CIVIL ENGINEERING
FACULTY OF ENGINEERING AND TECHNOLOGY
JADAVPUR UNIVERSITY
KOLKATA – 700032**

MAY 2014

FACULTY OF ENGINEERING & TECHNOLOGY
DEPARTMENT OF CIVIL ENGINEERING
JADAVPUR UNIVERSITY
Kolkata, India

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Supervisor

(Dr. Dipankar Chakravorty)

Professor

Department of Civil Engineering

Jadavpur University, Kolkata

Date:

Head of the Department

(Dr. Saroj Mandal)

Professor

Department of Civil Engineering

Jadavpur University, Kolkata

Date:

Dean

(Prof. Sivaji Bandyopadhyay)

Faculty of Engineering and Technology

Jadavpur University, Kolkata

Date:

CERTIFICATE

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To My Parents

and Guide

Prof. Dipankar Chakravorty

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Date:

Place: Jadavpur University, Kolkata

Signature of the Candidate

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LIST OF CONFERENCE PUBLICATIONS

Title	Conference Name	Authors	Place, Year
Progressive failure analysis of laminated composite shells - A review	International Conference on Industrial Engineering Science and Applications (IESA 2014)	Jayashree Sengupta, Dr. Dipankar Chakravorty	NIT Durgapur, West Bengal , April 2-4 , 2014
Failure aspects of composite cylindrical shell roofs	1st International Conference on Mechanics of Composites	Jayashree Sengupta, Dr. Dipankar Chakravorty	Stony Brook University , Long Island , New York , USA , June 8- 12 , 2014

LIST OF NOTATIONS

Symbols used in expressions throughout the text are explained in detail immediately after their first appearance. Nevertheless, some generally applied notation rules and common symbols are given below.

A_0-A_7	Constant terms of displacement polynomial.
a, b	Length and width of shell in plan
D	Flexural rigidity matrix of the laminate.
$\{ d_e \}$	Element displacement vector
$\{ d \}$	Global displacement vector.
E_{11}, E_{22}, E_{33}	Modulus of elasticity along the directions 1, 2 and 3.
1,2 and 3	Local co-ordinates of a lamina.
G_{12}, G_{13}, G_{23}	Shear modulus of a lamina in 1-2, 1-3, and 2-3 planes corresponding to the local axes of that lamina.
c	Rise of cylindrical shell.
h	Shell thickness
M_x, M_y	Moment resultants per unit length of cylindrical shell.
M_{xy}	Torsion resultant per unit length of cylindrical shell.
N_1 to N_8	Shape functions for first to eight nodes of an element respectively.
N_x, N_y	In-plane normal force resultants per unit length in X and Y directions.
N_{xy}	In-plane shear force resultant per unit length of shell.
n	Number of plies in a laminate
Q_x, Q_y	Transverse shear resultants per unit length of shell.
q	Intensity of uniformly distributed load.
u, v, w	Translational degrees of freedom along X, Y and Z direction respectively.
w	Transverse displacement in cm.
X, Y and Z	Global co-ordinates of the laminate.
Z_k, Z_{k-1}	Top and bottom distance of the kth ply from mid-plane of a laminate.
α, β	Rotational degrees of freedom about Y and X axis respectively.
F_x	Overall Longitudinal Strength

F_Y	Overall Transverse Strength
F_S	In-Plane shear strength
$F_{S_{XY}}, F_{S_{YZ}}, F_{S_{XZ}}$	Shear strength in XY, YZ and XZ plane respectively
F_{XT}, F_{YT}, F_{ZT}	Tensile strength in X, Y and Z direction respectively
F_{XC}, F_{YC}, F_{ZC}	Compressive strength in X, Y and Z direction respectively
$\sigma_1, \sigma_2, \sigma_3$	Normal stresses in X, Y and Z direction respectively
$\tau_{12}, \tau_{23}, \tau_{13}$	Shear stress in XY, YZ and XZ plane respectively
$\tau_{12}^u, \tau_{23}^u, \tau_{13}^u$	Ultimate Shear stress in XY, YZ and XZ plane respectively
ϵ_1, ϵ_2	Strain along X and Y direction respectively
ϵ_{12}	Shear strain
$\epsilon_1^u, \epsilon_2^u$	Ultimate Strain along X and Y direction respectively
ϵ_{12}^u	Ultimate Shear strain
$\epsilon_{XT}, \epsilon_{XC}$	Allowable normal strain strengths of a lamina along the fiber direction in tension and compression respectively
$\epsilon_{YT}, \epsilon_{YC}$	Allowable normal strain strengths of the matrix along the perpendicular to the fiber direction in tension and compression respectively.
$\epsilon_{S_{XY}}, \epsilon_{S_{YZ}}, \epsilon_{S_{XZ}}$	Shear strain strengths of a lamina in 2-3, 1-3 and 1-2 planes respectively.
θ	Angle of lamination with respect to the X axis of the cylindrical shell.
ν_{ij}	Poisson's ratio which characterizes compressive strain along x_j direction produced by a tensile strain applied in x_i direction.
ξ, η, ζ	Natural co-ordinates of isoparametric elements.

ABSTRACT

Shell structures with their feasible configurations enjoy industrial preference as a roofing entity for large column free areas. Design and construction of shell structure is an old subject to structural engineers. But gradually as technology developed, the engineers started using improved material to construct shells and more sophisticated mathematical tools to analyse the complex surface geometry. These parallel developments have paved the way to the laminated composite to be used as shell materials, which can be actually analysed by finite element approach. Laminated composite, an innate choice to different industrial sectors for its huge specific strength and stiffness, good weathering resistance, are now being extensively employed in civil engineering. This thesis presents a finite element method based relative and parametric study of progressive failure analysis of thin walled composite cylindrical shell structures typically subjected to transverse uniform loading. Different stress based failure criteria and material property degradation model are implemented in analysis. As case studies, progressive failure analyses of composite laminates with varying lamination, orientation, and boundary conditions are taken up to study the effect on the first ply failure and progression of failure and the ultimate failure. The failure loads and the point of initiation of failure are studied critically from engineering stand point. The areas of damage are correlated with deflection values through explicit equations and the selective points where non-destructive tests may be carried out to get an idea about the extent of damage without going into elaborate testing are also suggested.

Chapter 1

INTRODUCTION

1.1 General

The past few decades consummated comprehensive research apropos composite materials and thin walled structures. The former is now in use in every possible engineering quarter, be it marine, aerospace or civil; the latter is known for its innate versatility, structural competence and ease of construction. Properties like high strength, stiffness to weight ratio, prolonged fatigue life ascertains the composites as a superior material choice. Thin walled composite structures are the industrially chosen ones. The shell structures are best known for roofing units for large column free space. Highlighting the shell profile, many new forms are ceaselessly being reviewed and appended to the expanding glossary of the shells. Cylindrical shell however dominated the milieu, having a wide range of requirement based application. The best example that could be cited is Northlight cylindrical shell; used where ample distribution of light is required. Naturally, this soaring demand for the composites would likewise question the serviceability of the same.

On a macroscopic level, the failure analysis is rather sophisticated. A composite material undergoes transition in multiple phases. Upon the application of loads, the laminate undergoes stresses relating to the properties of the constituent phases, i.e., matrix, fibre, and interface-interphase. The first ply failure occurs in the weakest lamina, where stresses exceed the allowable strength modifying the material properties. At first-ply failure, redistribution of stresses occurs within the remaining laminae of the laminate. This does not necessarily mean that the whole of lamina has undergone failure; it purely indicates the initiation. The laminate will be termed as damaged with degraded properties. The constitutive relations are changed accompanied by reduction in stiffness. The stiffness of the failed lamina is discarded and the rest are considered to remain unaffected. The remaining laminae continue to take up load till the ultimate strength is reached. Failure initiation as well as propagation until eventual collapse of the structure assesses its durability, thus making first ply failure and progressive failure of prime concern to researchers.

1.2 Thesis Objective

A ply-by-ply progressive failure analysis is desired to be carried out on cylindrical shells subjected to transverse uniform loading. This thesis intends to present the damage assessment by the inclusion of different failure criteria viz. Maximum Stress Theory, Maximum Strain

Theory, Tsai Hill Theory, Tsai Wu Theory and Hoffmann Theory. Furthermore it allows for the identification of the failure zone, analyse the damage growth, and finally determine the ultimate strength of the composite laminate. This comprised of the stress analysis, the failure analysis and a property-degradation model for the damaged lamina. A parametric study, reflecting the effect of orientation of fibers in a lamina, stacking sequence, varying boundary conditions, on the strength of the laminate is carried out. The shell is subjected to 'varying' boundary conditions which includes, besides uniform clamped or simply supported boundary condition, two more edge conditions that have practically come across in the industry-

- (i) Two opposite edges is clamped and others are simply supported and
- (ii) Two adjacent edges are clamped and others are simply supported.

Also, it speculates through the various accessible literatures associated with the progressive failure of the composite laminated shells.

1.3 Document Overview

Chapter 2 presents the appropriate background necessary to accomplish this research. Topics covered include the historical background of thin shells, classification of the forms and geometry. It also shed some light on the analysis methodology of shell structures. A brief outline on the laminated composite is also provided. In Chapter 3, a literature survey of both first ply failure and progressive failure is presented; also the methods used to model progressive failure in concert with finite element analysis is reviewed. Chapter 4 studies the methodology of Progressive Failure Analysis in detail giving a step by step understanding of the process. Chapter 5 outlines the scope of the present work on cylindrical shells. Chapter 6 presents the finite element formulation of the cylindrical shell structure based on which a finite element code is developed which analyses out the first ply failure load, the subsequent failure progress leading to the ultimate failure load under uniformly distributed load. Elasticity relations, the stress-strain relations, the transformation of the axes system, etc. are also included. Also covered are the various failure theories used in operation. Chapter 7 includes the results and a discussion of the analysis dotting out the first fly failure load and the ultimate failure load and the failure progress throughout the laminate under varying boundary conditions and lamination. Chapter 8 presents the major conclusions point wise and indicates the future scope of the present work

Chapter 2

BACKGROUND OF STUDY

2.1 Brief History of Thin Shells

The second half of the twentieth century witnessed the structural designers' manifestation in pushing the confines of their imagination through their pioneering designs in thin shell structures. Even though at first these structures seemed drastic, their performance and efficiency in a wide range of shape and geometry offered enough evidence to convince engineers of their usefulness in solving structural design challenges such as stadiums, factories, chapels, and other large span column free structures. This enhanced confidence, thereby leading to an extensive shell building movement. These expertises practiced mostly in the 1950s and 1960s, bestowing structural engineers of today with a wealth of examples from which to learn about the design and behavior of thin shell structures [1].

Architectural thin shells referred to in this work are a modern development. The domes and cylindrical shaped structures of antiquity and the Middle Ages were thick and could only resist compressive loads. Notable well-known examples are the Agrippa's pantheon- the oldest standing domed structure in Rome; St Peter's Cathedral in Rome, a renaissance structure whose dome dating back to the 1590s was designed by Michelangelo, St Paul's cathedral in London designed by Sir Christopher Wren was built about 300 years ago and the Taj Mahal built in the seventeenth century by the Mogul Emperor Shah Jahan. The first modern architectural shell is generally credited to that built by the Zeiss optical company in Austria in the 1920s. In the United States, shells were extensively studied by the aircraft industry in the 1930s. In 1933, Donnell, an aeronautical engineer, formulated the general equations for cylindrical shells, including both bending and membrane actions. Eduardo Torroja of Spain is credited for the systematic engineering study of *architectural* shells in the 1930s. Torroja passed on the legacy of combining geometry, mathematical formulas, and a form-finding approach to Candela, who continued pushing the field of shell design forward. It was the work of Felix Candela in Mexico that ignited the sudden surge of popularity of shells in the 1950s.

Although advancement in the field of thin concrete shells slowed considerably after the 1960s and early 1970s, it did not stop altogether. With the passage of time, shells were no longer restricted to roofing but different other application of shells came into function and made equally significant contribution to the development of other branches of engineering. The following gives a brief outline to the various applications-

- *Architecture and Building.* The development of masonry domes and vaults in the middle Ages made possible the construction of more spacious buildings.
- *Power and chemical engineering.* The development of steam power during the Industrial Revolution depended to some extent on the construction of suitable boilers. These thin shells were constructed from plates suitable formed and joined by riveting. More recently the use of welding in pressure vessel construction has led to more efficient designs.
- *Structural engineering.* An important problem in the early development of steel for structural purposes was to design compression members against buckling. A striking advance was the use of tubular members in the construction of the Forth railway bridge in 1889: steel plates were riveted together to form reinforced tubes as large as 12 feet in diameter.
- *Vehicle body structures.* The construction of vehicle bodies in the early days of road transport involved a system of structural ribs and non-structural paneling or sheeting. The modern form of vehicle construction, in which the skin plays an important structural part, followed the introduction of sheet-metal components, preformed into thin doubly curved shells by large power presses, and firmly connected to each other by welds along the boundaries.
- *Composite construction.* The introduction of fiberglass and similar lightweight composite materials has impacted the construction of vehicles ranging from boats, racing cars, fighter and stealth aircraft, and so on. The exterior skin can be used as a strong structural shell.
- *Miscellaneous Examples.* Other examples of the impact of shell structures include water cooling towers for power stations, grain silos, armour, arch dams, tunnels, submarines, and so forth.

2.2 Shells Classification and Geometry

Shells can be singly curved (e.g., cylinders and cones) or doubly curved (e.g., sphere or hyperbolic paraboloid). Paraboloid is a shell of revolution made by revolving a parabola about its axis. A hyperbola produces a hyperboloid of two sheets when rotated about its axis of symmetry, and a hyperboloid of one sheet when rotated about the common axis between its two parts. The latter is often used for cooling towers, because it can be formed of straight lines. Another doubly curved shape shaped of straight lines is the conoid; a straight line travels along another straight line at one end and a curve at the other end.

Shells of translation are generated by translating one curve along another. A circle-translated tangent to a straight line generates a cylinder, and if translated along another circle produces a torus. Hyperbolic paraboloids (hypar) are doubly curved surfaces with negative curvature. A hypar can be generated by lifting one corner of a square shape. A hypar can also be generated by translating a convex parabola along a concave one. If the convex parabola had been

translated along another convex parabola instead of the concave parabola, it would have produced an elliptic parabola with a positive curvature.

A simple transform in the geometric parameters can result in a very different shape with wholly different structural behavior.

2.3 Analysis of Shell Structures

The structural analysis of shells has had a long and difficult history. Shells were developed and accomplished fame even before the ready availability of computers and the FE method. Thus the designers went through considerable effort to verify their designs. Model analysis where Plexiglas models, or cementitious models, were strain gauged and loaded with weights, was laborious, expensive, and impractical for testing various trial shapes.

Numerically shells may be analysed either by linear elastic analysis based on theory of elasticity (which is hardly done due to complex computational procedures for assessment of deflection and stresses) or a number of numerical methods which includes

- Finite difference method
- Energy methods, such as Rayleigh's method, Rayleigh-Ritz method, Galerkin method, etc.
- Dynamic relaxation method
- Finite element method.

Among the above mentioned, the finite element method is considered to be the most functional due to its ability to cater to any arbitrary geometry, loading and boundary conditions. Researchers in different areas are engaged in analysis of composite plates and shells by finite element approach. The present study also pertains to FE method to analyse laminated cylindrical shell.

2.4 Laminated Composite

Combination of two or more physically or chemically distinct, insoluble in each other, yet rightfully arranged (or dispersed phases of the same) with an interface separating them are called composite materials. The characteristics of the composite materials are not depicted by any of its components individually. Composite materials can be of various types. Each ply (or lamina) consists of fibers oriented in the required direction, blended with the resin material. Many such laminae are bonded together to get the required structures. Fibers and resin materials determine the strength and stiffness of the composite materials [2]. Different fibers are in use for structural applications. The most commonly used fibers are made of glass, carbon and Kevlar. The resin materials are essentially bonding material, which binds the fiber together. Many types of resin materials, such as epoxy resin material, metal resin material

and ceramic resin materials are currently in use- the most commonly being epoxy resin, which can be categorised under polymer resin material.

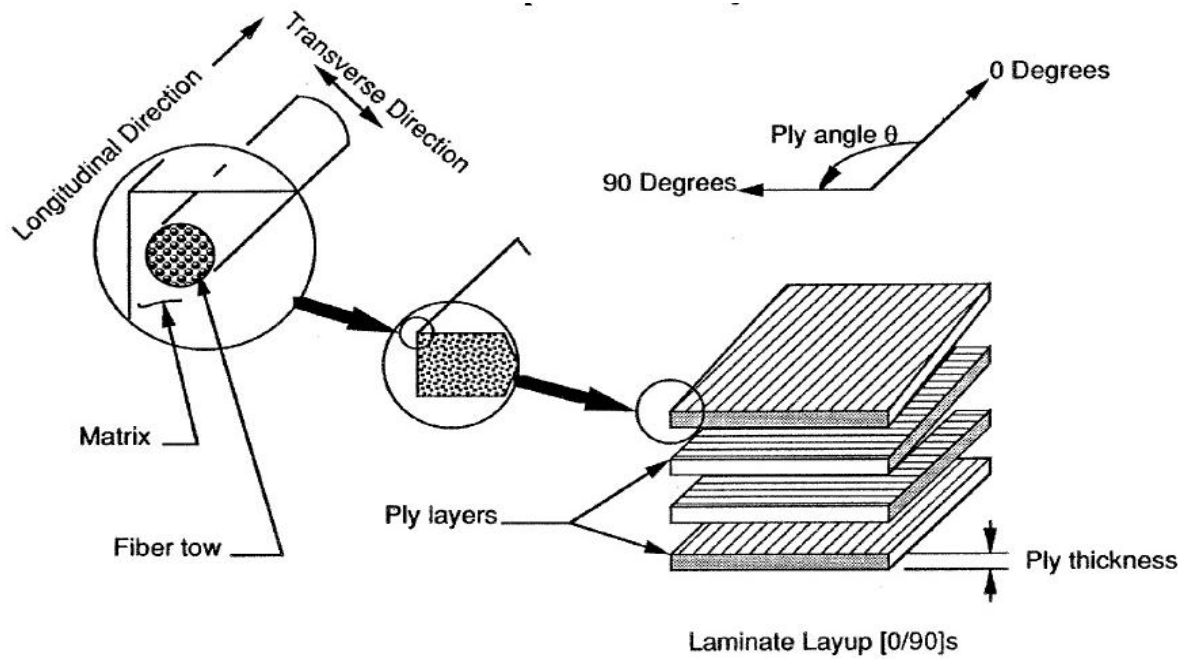


Fig. 2.1 Layout of Laminated Composite.

Formulation of a numerical model for laminates requires development of a proper constitutive model. This requires methods to estimate the material constants for the laminates starting from the properties of the constituent parts. In general, we need 4 material constants for 2-D characterisation and 9 material constants for 3-D characterisation of laminates. Once the material constants are determined, the constitutive law is established at the lamina level, the necessary transformation to appropriate coordinates system is done as in metals. The fiber orientation in the lamina is normally different from that of the global direction, and hence the constitutive law in local coordinate system is transformed to the global coordinate system.

Unlike metals which are isotropic in nature the composite materials exhibit anisotropic or orthotropic properties. In composite structures, failure is a complex process and can be an individual or a multitude of phenomena- be it fiber-breaks, matrix crack, delamination, debonding, or a whole lot of it. Typical mechanisms of failure causing failure initiation are generally dependant on the type of loading, geometry, material properties, condition of manufacture, boundary conditions, weather conditions etc. Since, composite materials exhibit directional properties, their applications and failure conditions should be properly examined and in addition to this, robust computational tools have to be exploited for the design of structural components for efficient utilization.

Chapter 3

LITERATURE SURVEY

3.1 General

The uses of shell structures and related research have a long history and the literature that has accumulated in this area is vast. Relevant literature regarding the progressive failure analysis of composite structures is briefly reviewed in this section. The review is presented in a chronological manner as far as possible.

3.2 Literature Review

Tsai and Wu [3] were the first to present that an operationally simple strength criterion cannot possibly explain the actual mechanisms of failures. Failures are but a multitude of independent and interacting mechanisms. They made use of strength tensors fulfilling the invariant requirements of coordinate transformation; interaction terms were treated as independent components and the difference in strengths owing to positive and negative stresses were accounted for making it way too improved than the Hill criterion wherein the interactions were not independent of each other. Previously, most initial failure analyses was concerned with the in-plane loading cases, perhaps because of being more governing in laminated structural elements. But Turvey [4] focused his study where flexural load dominated limiting his research to high modulus GFRP and CFRP and cross-ply symmetric configurations. He considered Tsai-Hill failure criterion as it is in good correlation for the GFRP laminates; expressed both the deflection surface and lateral pressure deflection in Navier double series form. Though his research was unrestricted on grounds of loading, he however chose three cases viz. uniformly distributed load, patch loading and hydrostatic loading varying linearly.

Reddy and Pandey [5] studied the first ply failure for the laminated composite plates, subjected to in-plane and/or transverse loading, the first order shear deformation theory and a tensor polynomial failure criterion (emphasis was laid on maximum stress, maximum strain, Tsai-Hill, Tsai-Wu and Hoffman's criteria) to predict failure at elemental Gauss points. They inferred that when the laminate was subjected to in-plane loading all the failure criteria were capable of predicting failure. However when the same was subjected to transverse loading, maximum strain and Tsai-Hill happened to have different results. The procedure that they depicted was an iterative one; once the first-ply failure had occurred the load was reduced by 20% and the process went on till the difference between any two successive failure loads was

less than 1%. They extended their research on post first ply failure emphasizing on the progressive analysis [6].

Tolson and Zabaras [7] studied progressive failure in laminated composite plates implementing seven degrees of freedom (three displacements, two rotations of normal about the plate mid-plane, and two warps of the normal). The in-plane stresses were calculated from the constitutive equations, but the transverse stresses were assessed from the three-dimensional equilibrium equations. The normal stress distributions were calculated at each Gauss points. Thereafter the stresses were employed in five failure criteria (maximum stress, Lee, Hashin, Hoffman, and Tsai-Wu) and were checked against whether failure had occurred or not. The Lee criterion gave the best results.

Prior to this, Engblom and Ochoa [8] carried out PFA till last ply failure; stiffness decrement and damage growth followed standard laminate analysis. Analysis was carried out on a plate subjected to uniaxial tension and four-point bending. Chang and Chang [9] developed the progressive damage analysis of notched laminated composites subjected to tensile loading. The progressive failure method used a nonlinear FEA using the modified Newton-Raphson iteration scheme to work out the state of stress in a composite plate.

Chang and Lessard [10] studied the damage in laminated composites containing an open hole subjected to compressive loading, wherein the in-plane response of the laminates from initial loading to final collapse was studied. A geometrically non-linear formulation based on finite deformation theory was used. Reddy and Reddy [11] computed linear and non-linear first-ply failure loads of composite plates for different load cases and edge conditions. The linear loads results varied by a maximum of 35% and for non-linear loads it was 50%. Besides, this difference was much large for thin laminates subjected to transverse loading and quite small for thick laminates subjected to in-plane loadings. They extended their work [12] to nonlinear progressive analysis using the Generalized Laminate Plate Theory (GLPT) of Reddy applying a new stiffness reduction format.

Kim and Hong [13] studied macroscopic failure models evaluating the stiffness changes employing shear lag factor and fiber bundle failure. Tan and Perez [14] studied progressive failure of laminated composites with holes, subjected to compressive loading predicting the extent of damage at any level of loading. Results showed good assessment with the experimental results. Tan [15] had previously developed progressive failure model of laminated composites subjected to in-plane loading considering the environmental impacts (thermal and hygroscopic stresses). Kam and Sher [16] studied progressive failure of centrally loaded laminated composite plates. The Ritz method, with geometric non-linearity, in the Von Karman sense, was used to construct the load displacement behaviour.

Cheung et al. [17] presented a PFA of composite plates by the finite strip method based on higher order shear deformation theory and Lee's failure criterion. Sahid and Chang [18] developed a model for predicting the effects of matrix crack induced accumulated damage on the in-plane response of laminated composites under tensile and shear loads. Echaabi et al. [19] presented a theoretical and experimental study of damage progression and failure modes of composite laminates under three point bending. Kim et al. [20] carried out PFA to predict the failure strengths and failure modes (tension, shear-out and bearing) which were judged against experimental data, of pin-loaded laminated composite plates using the penalty finite element method. Hashin's failure criteria was performed for damage evaluation in the laminates.

Gummadi and Palazotto [21] performed PFA of composite cylindrical shells with large rotations based on Lagrangian approach and the due changes in the constitutive relations were discussed; considered failure modes of matrix cracking, fiber breakage and delamination; damage progression followed maximum stress criterion. The stiffness matrix was determined based on Green's strain and 2nd Piola Kirchhoff stresses. Green's strain was transformed to Almansi strain which was further transformed into material axis system and was used to determine the Euler stresses. These Euler stresses were the key to failure determination. Padhi et al. [22] presented their detailed study on progressive failure and ultimate collapse of laminated composites using Hashin and Tsai-Wu's failure criteria. They made use of Newton-Raphson Method with a force and moment residual convergence of 0.5% and displacement correction convergence of 1%. The model was capable of assessing type and extent of damage all throughout.

Spottswood and Palazotto [23] determined the response together with material failure of a thin curved composite shell resisting transverse loading, incorporated simplified large displacement/rotation (SLR) theory and compared the results with previously published available data; failure criteria being Hashin's. Pal and Ray [24] carried out PFA under transverse static loading in linear elastic range distinctively for both antisymmetric and symmetric angle ply laminates. Knight Jr. et al. [25] reviewed the overall computational issues and requirements for performing PFAs using STAGS (Structural Analysis of General Shells) for solving non-linear quasi-static structural response problems including special details. Prusty [26] studied unstiffened and stiffened composite panels under transverse static loadings in the linear elastic range. Eight-noded isoparametric quadratic elements with three-noded curved beam elements were modeled and checked against various failure theories. Ultimate ply failure loads for the stiffened panels with cross-ply and angle-ply laminations in the shell was analyzed.

Akhras and Li [27] proposed a spline finite strip method PFA of thick composite plates based on Cho's higher order zigzag laminate theory and Lee's failure criterion. The transverse shear stresses were obtained directly from the constitutive equations; the shear correction factor

was not required as for the first-order shear theory. The procedure involved an incremental load analysis through modified Newton-Raphson method and standard PFA method showing good agreement with the 3D finite element solutions. Las and Zemčík [28] proposed a PFA model of unidirectional composite panels using Puck's fiber and inter-fiber criteria demonstrating the results with examples of tensile tests of single-ply panels.

Tapered laminated plates under the action of uniaxial compression were investigated by Ganesan and Liu [29] predicting different failure loads and the associated displacements, locations and modes. The influences of the tapered configuration, lay-up configuration, and fiber orientation were also the concern of the study. Earlier, Ganesan and Zhang [30] had conducted a detailed investigation of the progressive failure of uniform thickness laminates subjected to uniaxial compression. Singh et al. [31, 32, 33] also studied the progressive failure of uniform thickness plates subjected to different loads. Progressive damage analysis methodology for stress analysis of composite laminated shells using finite strip methods based on Mindlin's plate-bending theory were addressed by Zahari and El-Zafrany [34], where the non-linear equations were derived using the tangential stiffness matrix approach; validation was done by comparing the results with analysis in ABAQUS.

Sandwich composite panels under quasi-static impact were investigated by Fan et al. [35]. Hashin's and Besant's criteria were checked for different failure mechanisms. Ply discounting method was employed as the strategy for material degradation. Ahmed and Sluys [36] designed a computational model presenting a mesoscopic failure model studying matrix cracking, delamination and the combined effect. A mesh independent matrix cracking was modeled with discontinuous solid-like shell element (DSLS); delamination was presented by a shell interface model. Besides material nonlinearities, the numerical model simulated geometrical nonlinearities.

Anyfantis and Tsouvalis [37] studied post-buckling progressive and final failure response of stiffened composite panels using ANSYS. The intralaminar, fiber and matrix failure modes in compression and tension were addressed using a combined framework of Hashin's and Tai-Wu failure. The PFA method included intralaminar failures that stimulated material degradation of the failed layers. Pietropaoli [38] too worked on progressive failure of composite plates using a constitutive model implemented in ANSYS. Standard ply discount technique was used and the onset and progression of damage was observed and the results were validated against experimental results.

Bogetti et al. [39] studied the nonlinear response and progressive failure of composite laminates under tri-axial loading. A program was build and executed in MATLAB; this analyzed and displayed the failure envelope. Philippidis and Antoniou [40] computed a PFA model for glass/epoxy composite giving an extensive comparison between numerically calculated stress-strain response up to failure and experimental data. Cárdenas et al. [41]

presented a reduced-order FE model suitable for PFA of composite structures under dynamic aeroelastic conditions based on a Thin-Walled Beam theory predicting both onset and propagation of damage.

Ellul and Betts [42] studied PFA applying them to Fiber-Reinforced Composite Plates subjected to out-of-plane bending. They studied influence of different boundary conditions of the plates on the ultimate ply failure load and concluded that the predominant failure mechanism is the failure of matrix and after the redistribution of stresses, no consecutive failure due to fiber or fiber-matrix failure occurs in the lamina. Liu and Zhang [43] investigated low-velocity impact responses and impact-induced damages on the stiffened composite laminated plates based on the progressive failure model and layerwise/solid-elements method and implemented Hashin 3D failure criterion.

3.3 Critical Discussion

A close review of the literature shows that the large numbers of researchers are now working on different aspects of composite structural elements including shell structures. Recently a number of research reports have appeared in the referred journals which deal with different failure criteria of composites for performance evaluation of those. It is found that such studies on beams and plates quite enjoyed the focus; however areas concerning shell structures are really scarce and yet to be quarried. Hence failure analyses of composite shells need careful attention.

Chapter 4

PROGRESSIVE FAILURE ANALYSIS

4.1 General

To arrive at the safe design, the designer must be aware of the limits of the material. Failure in composites starts with the failure of the weakest lamina and may propagate leading to progressive failure of the structure. For reliability assertion, the predictions of the failure process of laminated composite structures and the maximum loads that the structures can withstand before failure occurs has thus become an important topic of research.

The failure of a lamina in a laminate does not signify failure, as the failure of the laminate is progressive. Each lamina in a laminate is assumed to be in a state of plane stress. Stresses in each lamina are evaluated separately by transforming the stresses in the principal material axis direction and applying suitable failure criteria. The analysis of failure of a laminate is therefore more complicated than a single lamina. Various parameters such as orientation of fibres in a lamina, stacking sequence, fabrication process and others dictate the strength of the laminate.

A laminate may fail in one of the following types of failure modes:

- first ply failure (FPF),
- ultimate ply failure (UPF) and
- interlaminar failure.

FPF is defined as the failure of the laminate with the failure of the weakest layer. ULF is defined as the failure of the laminate when ultimate load capacity is reached following failure of all plies. Inter-laminate failure is defined as the failure resulting from the separation of the adjacent layers though the individual lamina remains intact leading to delamination.

The first ply failure is similar to treating the laminate as having reached the ‘yield stress’ and the ultimate laminate failure is analogous to the ultimate stress criterion. In FPF, higher safety factors may be considered in the analysis, as the whole approach is rather conservative. In UPF analysis, however, more precision is needed about the load distribution. In primary structures, the working loads are generally kept below that producing FPF.

4.2 Progressive Failure Analysis

As it was stated before, to carry out Progressive Failure Analysis (PFA), it is necessary to determine the First Ply Failure (FPF) of the laminate for the simulation of the failure progression from the beginning of the failure to the ultimate failure load level. The material properties of the lamina, mechanical loading i.e. forces and moments, and layer orientation are read. Based on the inputs, the stresses at each Gauss point within individual lamina were evaluated and are verified with the failure criteria to check any possible failure. If any failure had occurred, the material properties at that point were modified with accordance to the observed failure mode, and the stresses were recalculated at FPF load, using a property degradation technique-

(A) Total-Ply Failure Method and

(B) Partial-Ply Failure Method.

Thereafter, a check is performed to see whether the second ply fails at FPF load, if not, a load increment is performed and with the reduced stiffness the process continues till the ultimate failure load occurs and at that stage convergence of stresses cannot be achieved numerically . A typical PFA is demonstrated in Fig. 4.1.

An incremental load is applied and then iteration is undertaken until a converged solution has been achieved. Once the converged solution has been achieved, after the stress recovery a failure criterion is invoked to detect local lamina failure and determine the failure mode. If no failure is detected at the particular load level, the load is incremented again and the whole process of establishing the equilibrium, stress recovery and check of the failure criterion is repeated at the current load level. If failure is detected at a particular load level, then material degradation or damage models are needed in order to determine new estimates of the local material properties and propagate the failure. After the degradation of the material properties of the damaged layer, a finite element analysis is conducted at the same load level without incrementing the load. Since the material properties are degraded locally due the failure induced, equilibrium must be re-established. Once the equilibrium is re-established, stress recovery and failure checks are performed as before. The loop continues. Because of the stiffness loss due to the accumulation of the damages, it turns out that at a certain load level failure propagates without incrementing the load until equilibrium cannot be established. Usually, this load level is referred to as ultimate load.

4.3 Material Degradation Models

4.3.1. Total Ply Failure Method

On reaching the failure, the strength and stiffness of the failed ply is totally reduced to zero. This implies that if the ply undergoes matrix failure, it is no longer able to carry load in fibre

direction, which, may not be the case. Thus this method somehow underestimates the laminate strength.

4.3.2. Partial Ply Failure Method

In this approach, the failure mode is taken into account. If the ply fails due to fibre failure, the stiffness of the failed ply is reduced to zero. However, if it is a matrix controlled failure or shear failure, the longitudinal modulus retains its value but the transverse and shear modulus are set to zero.

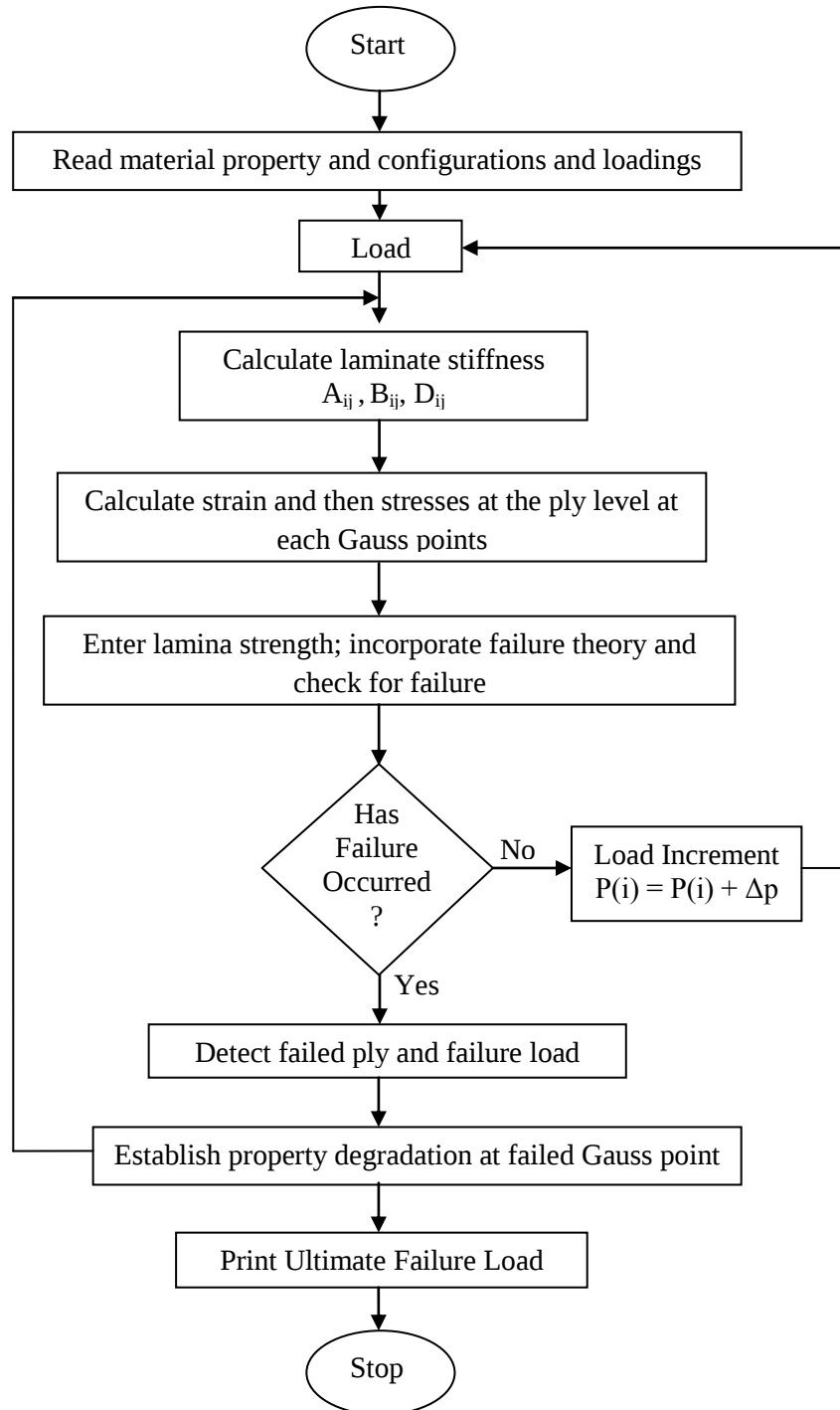


Fig. 4.1 Schematic Diagram of Progressive Failure Analysis.

Chapter 5

SCOPE OF PRESENT STUDY

5.1 General

A review of the literatures that exists on shell research is presented in the Chapter 3 with emphasis on the developments that took place in the last two to three decades. With the data pulled together from the literature review and keeping in view the voids in the volume of accumulated literature as indicated in the critical discussion, the actual scope of the present investigation is outlined in this chapter.

5.2 Present Scope

Cylindrical shells are very much industry friendly from construction as well as aesthetic point of view. Studies on cylindrical shells have been performed by many researchers. The research on cylindrical shells is still on as evident from the studies on multi-objective optimization of laminated cylindrical shells by Umut Topal [41], elastic buckling of stiffened cylindrical shells with rings and stringers made of functionally graded materials subjected to axial compression loading, presented by Najafizadeh, Hasani, and Khazaeinejad [43] and the like. A revolution occurred in the field of shell research when composites were introduced as the construction material of different shells. Due to many positive qualities of composite materials various researches have carried out studies on composite shells.

From the discussion presented in the preceding chapters it is clear that researchers have channelized their studies on composite materials from various perspectives from time to time. Despite having complexities in modeling composites mathematically, a large number of research works have been done using the finite element approach. One of the most important among those studies is that on failure. Different failure theories were proposed at different times and consequently they were modified and improved.

A designer can efficiently apply composite shells in industry, only when he has a very good idea on the strength of the material used. Though first ply failure does not signify total failure; it is very important from strength perspective. If first ply failure occurs and remains unobserved, it may lead to devastating failure suddenly. Failure criteria can be used to determine first ply failures as well as further progressive failure, or failure after the first ply failure has occurred. Progressive failure analysis incrementally reduces the strength of the composite structure using various techniques. As prelude to earlier, composite materials, can

retain significant strength even after first ply failure. It is therefore beneficial to analyze composite structures both for first ply failure and for the failures that follow. For the aforementioned reasons, a means of analyzing progressive failure is extremely important in the analysis of composite structures. There are many avenues available to engineers to aid in this investigation. Some methods use statistical analyses, micromechanics or fracture mechanics approaches to predicting lamina damage progression. Finite element analysis provides engineers a ready platform for the study of failure in composite materials. Failure criteria have been included in many finite element methods used to analyse composite structures for both first ply failure and last ply failure.

A mathematical formulation using finite element as a numerical tool is presented in Chapter 6. An eight-noded curved quadratic isoparametric shell element is developed for analysing the linear first ply and progressive failure characteristics of laminated cylindrical shells. Apart from solution of benchmark problems to establish the validity of the present approach, a number of other problems of composite cylindrical shells subjected to transverse uniformly distributed load are solved. Four different stacking sequences of graphite-epoxy composite and four different types of boundary conditions are considered. The results are discussed in details and the important conclusions are highlighted.

Chapter 6

MATHEMATICAL FORMULATION

6.1 General

Based on the thesis objectives and present scope, a mathematical formulation is presented in this chapter, based on which a finite element code is developed which can effectively be utilized to calculate the first ply failure load, the subsequent leading to the ultimate failure load of composite cylindrical shells under uniformly distributed load.

6.2 Finite Element Method

In finite element method the cylindrical shells under consideration is sub-divided into a series of smaller regions in which the governing differential equation is numerically solved. By assembling the set of equations for each region, the behavior over the entire problem domain is determined. Each region is referred to as an element and the process of subdividing a domain into a finite number of elements is referred to as discretization. Elements are connected at specific points, called nodes.

6.3 Shell Element

The shell surface considered for the present study is cylindrical shell (Fig. 6.1) with singly curved and synclastic surface having curvatures $\frac{1}{R_Y}$ only ($\frac{1}{R_{XY}} = \frac{1}{R_X} = 0$ for cylindrical shell). A singly curved thin shallow shell of uniform thickness 'h' made of homogeneous, laminated composite, linearly elastic material is considered. The projection of the shell on the XY plane is a rectangle of dimension 'a' and 'b'. The equation of the middle surface of the shell, referred to the Cartesian coordinate system (x, y, z) is expressed as:

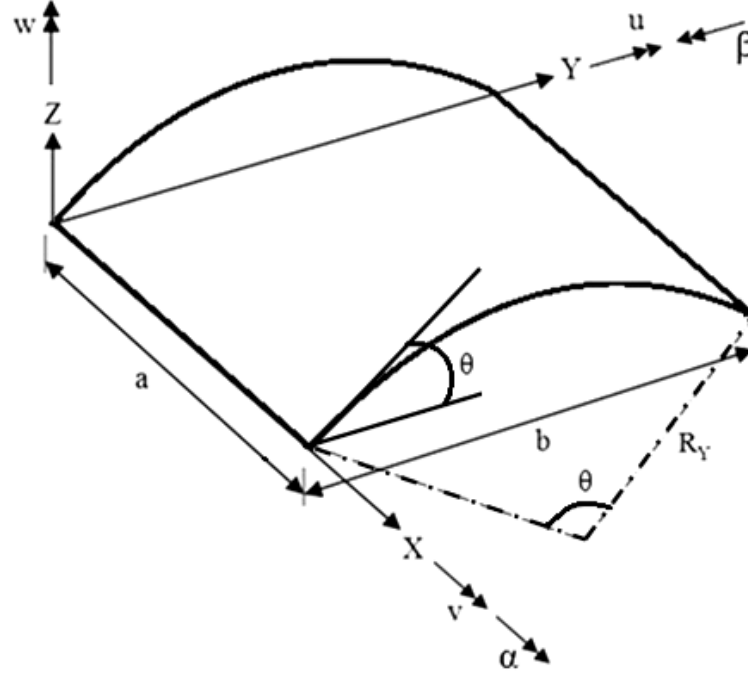
$$z = f(x, y) \quad (6.1)$$

A shell is characterized as shallow if any infinitesimal line element of its middle surface is approximated by the length of its projection on the XY plane. This implies that

$$\left(\frac{\partial z}{\partial x}\right)^2 \ll 1 \quad \left(\frac{\partial z}{\partial y}\right)^2 \ll 1 \quad \frac{\partial^2 z}{\partial x \partial y} \ll 1 \quad (6.2)$$

Moreover, the lateral boundary of a shallow shell is approximated by its projection on the XY plane with regard to its boundary conditions. According to Vlasov (1958), the above conditions are practically satisfied for shells with a rise to span ratio less than 1/5 and the curvatures are approximately represented as:

$$\frac{1}{R_X} = \frac{\partial^2 z}{\partial x^2}, \quad \frac{1}{R_Y} = \frac{\partial^2 z}{\partial y^2}, \quad \frac{1}{R_{XY}} = \frac{\partial^2 z}{\partial x \partial y} \quad (6.3)$$



Surface Equation:
$$z = \sqrt{R_Y^2 - (X \sin \theta - Y)^2} - X \cos \theta \cdot R_Y$$

Fig. 6.1 Surface of a cylindrical shell and degrees of freedom

6.4 Selection of Shell Element

In the finite element analysis the structure has to be discretized into a number of elements connected at the nodal points. The element shall be such that it can properly define the behavior of the structure. In the present analysis an eight noded doubly curved quadratic isoparametric element is chosen having all three radii of curvature. In our present investigation our analysis is confined only at cylindrical shell, which has radius of curvature R_Y along y-direction only. The quadrilateral element has four corner nodes and four midpoint nodes as shown in Fig. 6.2. The isoparametric element shall be oriented in the natural coordinate system (ξ, η) and shall be transformed to the Cartesian coordinate system using the Jacobian matrix. The full surface of the cylindrical shell is discretized to account for its curvature. The order of numbering of elements and nodes over the plan area of a typical shell surface is shown in Fig. 6.3.

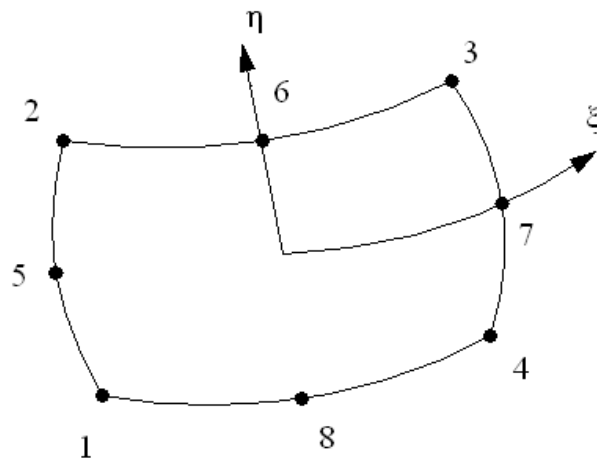


Fig. 6.2 The shell element with isoparametric coordinates

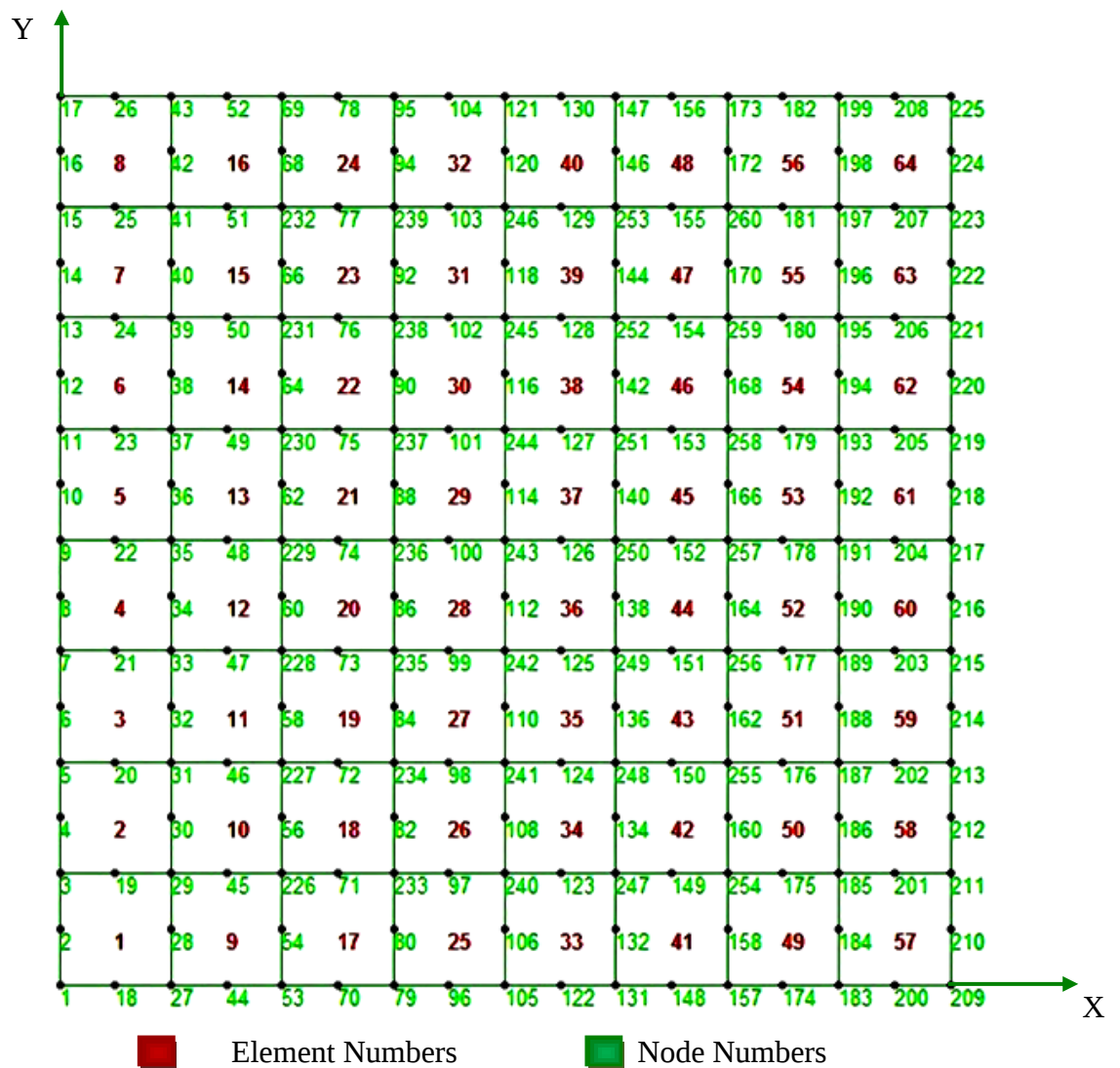


Fig. 6.3 A typical discretization of 8x8 mesh on plan area with element and node numbers

6.5 Selection of Shape Functions

The shape functions or the interpolation functions are polynomials of the natural coordinates (ξ, η) which relate the generalized displacements at any point within an element to the nodal values of the displacements. These are derived from an interpolation polynomial in terms of the natural coordinates so that the displacement fields are satisfactorily represented. For the analysis of thin shells where the final element is assumed to have mid-surface nodes only, the interpolation polynomial is a function of ξ and η and has the following form-

$$u(\xi, \eta) = A_0 + A_1 \xi + A_2 \eta + A_3 \xi^2 + A_4 \xi\eta + A_5 \eta^2 + A_6 \xi^2\eta + A_7 \xi\eta^2 \quad (6.4)$$

The shape functions derived from interpolation polynomial are as

$$\left. \begin{aligned} N_i &= \frac{1}{4} (1 + \xi \xi_i) (1 + \eta \eta_i) (\xi \xi_i + \eta \eta_i - 1) \quad ; \quad i = 1, 2, 3, 4 \\ N_i &= \frac{1}{2} (1 + \xi \xi_i) (1 - \eta^2) \quad ; \quad i = 5, 7 \\ N_i &= \frac{1}{2} (1 + \eta \eta_i) (1 - \xi^2) \quad ; \quad i = 6, 8 \end{aligned} \right\} \quad (6.5)$$

where, denotes the shape function at i th node having natural coordinates and ξ_i and η_i . The correctness of the shape functions is checked from the relations

$$\sum N_i = 1, \quad \sum \frac{\partial N_i}{\partial \xi} = 0, \quad \sum \frac{\partial N_i}{\partial \eta} = 0 \quad (6.6)$$

In an isoparametric formulation the generalized displacements and coordinates are interpolated from their nodal values by the same set of shape functions. Hence, the coordinates (x, y) of any point within an element are obtained as

$$x = \sum_{i=1}^n N_i x_i \quad y = \sum_{i=1}^n N_i y_i \quad (6.7)$$

where x_i and y_i are the coordinates of the i th node.

6.6 Selection of the Generalised Displacement Fields and Nodal Displacements

Any shell surface can be modeled by three dimensional solid elements. When the thickness dimension is considerably smaller than the other dimensions, the nodes along the thickness direction supply additional degrees of freedom than needed and hence are not preferred. When a two dimensional element is obtained by condensing the thickness direction nodes. The displacements of adjacent thickness direction nodes must be ensured to be equal to avoid

numerical difficulties. Thus five degrees of freedom including three translations (u, v, w) and two rotations (α, β) are attached to each node. The final element has mid-surface nodes only and a line in the thickness direction remains straight but not necessarily normal to the mid-surface after deformation. The directions of the generalised displacements are shown in Fig. 6.1.

The following expressions establish the relations between the displacement at any point with respect to the co-ordinates ξ and η and the nodal degrees of freedom.

$$\left. \begin{aligned} u &= \sum_{i=1}^8 N_i u_i & v &= \sum_{i=1}^8 N_i v_i & w &= \sum_{i=1}^8 N_i w_i \\ \alpha &= \sum_{i=1}^n N_i \alpha_i & \beta &= \sum_{i=1}^n N_i \beta_i \end{aligned} \right\} \quad (6.8)$$

The generalized displacement vector of an element is expressed in terms of the shape functions and nodal degrees of freedom as

$$[u] = [N] \{d_e\} \quad (6.9)$$

$$\text{i.e. } \{u\} = \begin{Bmatrix} u \\ v \\ w \\ \alpha \\ \beta \end{Bmatrix} = \begin{bmatrix} [N_i] & & & & \\ & [N_i] & & & \\ & & [N_i] & & \\ & & & [N_i] & \\ & & & & [N_i] \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \alpha_i \\ \beta_i \end{Bmatrix} \quad (6.10)$$

where,

$$\left. \begin{aligned} [N_i] &= [N_1 \quad N_2 \quad N_3 \quad \cdots \quad N_8], & [u_i] &= [u_1 \quad u_2 \quad u_3 \quad \cdots \quad u_8] \\ [v_i] &= [v_1 \quad v_2 \quad v_3 \quad \cdots \quad v_8], & [w_i] &= [w_1 \quad w_2 \quad w_3 \quad \cdots \quad w_8] \\ [\alpha_i] &= [\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \cdots \quad \alpha], & [\beta_i] &= [\beta_1 \quad \beta_2 \quad \beta_3 \quad \cdots \quad \beta_8] \end{aligned} \right\} \quad (6.11)$$

6.7 Strain Displacement Equations

The strain-displacement relations on the basis of improved first order approximation theory for thin shell are established as-

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \kappa_{xz} \\ \kappa_{yz} \end{Bmatrix} \quad (6.12)$$

where, the first vector is the mid-surface strain and the second vector is the curvature and are related to degrees of freedom as

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - \frac{2w}{R_{xy}} \\ \alpha + \frac{\partial w}{\partial x} \\ \beta + \frac{\partial w}{\partial y} \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \kappa_{xz} \\ \kappa_{yz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \alpha}{\partial x} \\ \frac{\partial \beta}{\partial y} \\ \frac{\partial \alpha}{\partial y} + \frac{\partial \beta}{\partial x} \\ 0 \\ 0 \end{Bmatrix} \quad (6.13)$$

The strain components of the equations (6.13) are to be considered together for generalized representation of the three dimensional strain fields and are expressed in the following form

$$\{\varepsilon\} = [H] \{d_e\} \quad (6.14)$$

$$\{d_e\} = \left\{ \frac{\partial u}{\partial x} \quad \frac{\partial u}{\partial y} \quad \frac{\partial u}{\partial z} \quad \frac{\partial v}{\partial x} \quad \frac{\partial v}{\partial y} \quad \frac{\partial v}{\partial z} \quad \dots \quad u \quad v \quad w \quad \alpha \quad \beta \right\}^T \quad (6.15)$$

and $[H]$ is a 8×20 matrix of the form given in Table 6.1

In the isoparametric formulation the displacements are interpolated from nodal values by the shape functions and hence the derivatives of the displacements are obtained with respect to the natural coordinates and then proper transformation technique is applied.

Table 6.1 Elements of $[H]$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
2	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
3	.	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	-1/R _{xy}	.	.
4	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.
5	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.
6	.	.	.	.	.	.	.	.	.	.	1	.	1	.	.	.	.	.	.	.
7	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	1	.
8	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	1

Thus the vector $\{d_e\}$ is expressed in terms of natural coordinates as

$$\{d_e\} = [J]^{-1} \{d_n\} \quad (6.16)$$

$$\{d_n\} = \left\{ \frac{\partial u}{\partial \xi} \quad \frac{\partial u}{\partial \eta} \quad \frac{\partial u}{\partial \zeta} \quad \frac{\partial v}{\partial \xi} \quad \frac{\partial v}{\partial \eta} \quad \frac{\partial v}{\partial \zeta} \quad \dots \quad u \quad v \quad w \quad \alpha \quad \beta \right\}^T \quad (6.17)$$

and $[J]$ is Jacobian matrix expressed as

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \end{bmatrix} \quad (6.18)$$

For thin shells, according to Gionku, the ratio of the thickness to minimum radius of curvature is less than 0.005. Hence, terms with first power of ζ in the Jacobian matrix is neglected.

From equation (6.17), it is evident that the vector $\{d_n\}$ is obtained multiplying the nodal displacement vector $\{d_i\}$ by a matrix $[\Lambda]$ containing the shape functions and derivatives with respect to the natural coordinates.

$$\text{Thus,} \quad \{d_n\} = [\Lambda]_{20 \times 5n} \{d_i\}_{5n \times 1} \quad (6.19)$$

where, n is the number of nodes in shell elements and $\{d_i\}$ is given by

$$\{d_i\} = \{u_1 \quad v_1 \quad w_1 \quad \alpha_1 \quad \beta_1 \quad \dots \quad u_n \quad v_n \quad w_n \quad \alpha_n \quad \beta_n\}^T \quad (6.20)$$

Combining the equations (6.14), (6.16) and (6.19) one gets

$$\begin{aligned} \{\varepsilon\}_{8 \times 1} &= [H]_{8 \times 20} \{d_e\}_{20 \times 1} \\ &= [H]_{8 \times 20} [J]_{20 \times 20}^{-1} \{d_n\}_{20 \times 1} \\ &= [H]_{8 \times 20} [J]_{20 \times 20}^{-1} [\Lambda]_{20 \times 5n} \{d_i\}_{5n \times 1} \\ &= [B]_{8 \times 5n} \{d_i\}_{5n \times 1} \end{aligned} \quad \left. \vphantom{\begin{aligned} \{\varepsilon\}_{8 \times 1} &= [H]_{8 \times 20} \{d_e\}_{20 \times 1} \\ &= [H]_{8 \times 20} [J]_{20 \times 20}^{-1} \{d_n\}_{20 \times 1} \\ &= [H]_{8 \times 20} [J]_{20 \times 20}^{-1} [\Lambda]_{20 \times 5n} \{d_i\}_{5n \times 1} \\ &= [B]_{8 \times 5n} \{d_i\}_{5n \times 1} \end{aligned}} \right\} \quad (6.21)$$

where $[B]$ is the strain displacement matrix.

6.8 Deriving Force-Strain Relationship of the Composite Material

Before discussing methods by which structural properties of a composite may be assessed, it is required to recognize that actual nature of fiber reinforced composite lamina is extremely complex and in order to produce micromechanical solutions, certain simplifying assumptions are required.

The basic assumptions are,

- Both the fiber and matrix is homogeneous and isotropic.
- The fiber, the matrix and the resulting composite exhibit linear elastic behavior.
- Perfect bond exist between fibers and matrices so that no slipping occurs at the surface.
- Fibers are regularly spaced, uniform and perfectly aligned.
- The matrix is free of voids.
- The lamina is at stress free state.

6.8.1 Elastic Properties of Unidirectional Lamina

6.8.1.1 Stress- Strain Relationship

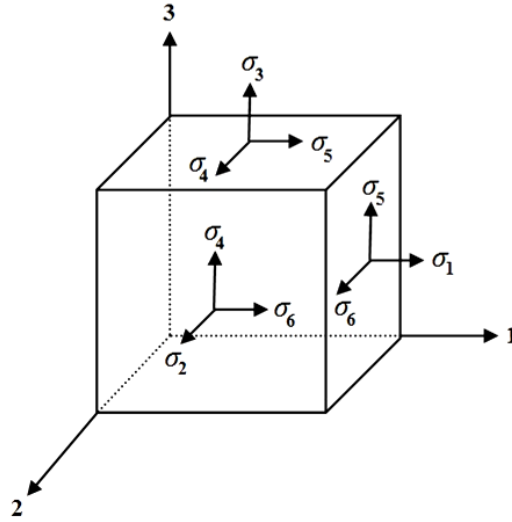


Fig. 6.4 Stress components in a three dimensional continuum

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} \Rightarrow \sigma_i = C_{ij} \varepsilon_j \quad (6.22)$$

where, ε_i = strain components corresponding to σ_i and (i, j=1,2,3,4,5,6).
and, C_{ij} = constitutive relationship matrix.

$$\text{Work done per unit volume of the continuum, } W = \frac{1}{2} \sigma_i \varepsilon_i \quad (6.23)$$

$$\text{Substituting } \sigma_i \text{ from 6.22 to 6.23, we get, } W = \frac{1}{2} C_{ij} \varepsilon_i \varepsilon_j \quad (6.24)$$

$$\text{Stress can be expressed as, } \sigma_i = \frac{\partial W}{\partial \varepsilon_i} = C_{ij} \varepsilon_j \quad (6.25)$$

$$\text{Differentiating again with respect to } \varepsilon_j, \text{ we get, } \frac{\partial^2 W}{\partial \varepsilon_i \partial \varepsilon_j} = C_{ij} \quad (6.26)$$

Similarly, if the differentiation is first performed with respect to ε_j and then with respect to ε_i , we get, $\frac{\partial^2 W}{\partial \varepsilon_j \partial \varepsilon_i} = C_{ji}$ (6.27)

So, we can see from equation 6.26 and 6.27 that $C_{ij} = C_{ji}$ and hence it is also established that the constitutive matrix for an anisotropic material is symmetric.

Now from the theory of elasticity compliance matrix can be written as

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{21}}{E_2} & \frac{-\nu_{31}}{E_3} & 0 & 0 & 0 \\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & \frac{-\nu_{32}}{E_3} & 0 & 0 & 0 \\ \frac{-\nu_{13}}{E_1} & \frac{-\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} \quad (6.28)$$

By inverting the above compliance matrix we get the constitutive relationship matrix of the anisotropic material as,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} \quad (6.29)$$

$$C_{11} = E_1 \left(\frac{1 - \nu_{23}\nu_{32}}{\Delta} \right), \quad C_{22} = E_2 \left(\frac{1 - \nu_{13}\nu_{31}}{\Delta} \right), \quad C_{33} = E_3 \left(\frac{1 - \nu_{12}\nu_{21}}{\Delta} \right)$$

$$C_{12} = E_1 \left(\frac{\nu_{21} + \nu_{31}\nu_{23}}{\Delta} \right) = E_2 \left(\frac{\nu_{12} + \nu_{32}\nu_{13}}{\Delta} \right)$$

$$C_{13} = E_1 \left(\frac{\nu_{31} + \nu_{21}\nu_{32}}{\Delta} \right) = E_3 \left(\frac{\nu_{13} + \nu_{12}\nu_{23}}{\Delta} \right)$$

$$C_{23} = E_2 \left(\frac{\nu_{32} + \nu_{12}\nu_{31}}{\Delta} \right) = E_3 \left(\frac{\nu_{23} + \nu_{21}\nu_{13}}{\Delta} \right)$$

$$C_{44} = G_{23}, \quad C_{55} = G_{31}, \quad C_{66} = G_{12}$$

$$\Delta = 1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{13}\nu_{31} - 2\nu_{12}\nu_{23}\nu_{13}$$

6.8.1.2 Derivation of Reduced Stiffness Matrix of an Individual Lamina

Now an individual lamina is very thin and any stress component in the transverse direction can be neglected, but in the present study transverse shear deformation is considered. So, for an individual lamina, $\sigma_{33} = 0$. Now, we are separating the in plane and out-of plane stress components and hence equation 6.29 results into,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ 0 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 \\ C_{21} & C_{22} & C_{23} & 0 \\ C_{31} & C_{32} & C_{33} & 0 \\ 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_6 \end{Bmatrix} \quad (6.30)$$

$$\text{and } \begin{Bmatrix} \sigma_4 \\ \sigma_5 \end{Bmatrix} = \begin{bmatrix} C_{44} & 0 \\ 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix} \quad (6.31)$$

Solving 6.30, yields,

$$\sigma_1 = \left(C_{11} - \frac{C_{13}^2}{C_{33}} \right) \varepsilon_1 + \left(C_{12} - \frac{C_{13}C_{23}}{C_{33}} \right) \varepsilon_2 = Q_{11}\varepsilon_1 + Q_{12}\varepsilon_2$$

$$\sigma_2 = \left(C_{12} - \frac{C_{13}C_{23}}{C_{33}} \right) \varepsilon_1 + \left(C_{22} - \frac{C_{23}^2}{C_{33}} \right) \varepsilon_2 = Q_{12}\varepsilon_1 + Q_{22}\varepsilon_2$$

$$\sigma_1 = C_{66}\varepsilon_6 = Q_{66}\varepsilon_6$$

Writing the above equations in a matrix form, we get

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} \quad \Rightarrow \quad \{\sigma\}_{1,2} = [Q]_{1,2} \{\varepsilon\}_{1,2} \quad (6.32)$$

Where,

$$Q_{11} = \left(\frac{E_1}{1 - \nu_{12}\nu_{21}} \right) \quad Q_{22} = \left(\frac{E_2}{1 - \nu_{12}\nu_{21}} \right) \quad Q_{12} = \left(\frac{E_1 \nu_{21}}{1 - \nu_{12}\nu_{21}} \right) \quad Q_{66} = G_{12}$$

The matrix $[Q]$ in equation 6.32 is termed as the reduced stiffness matrix of an individual lamina in the local co-ordinate system.

6.8.1.3 Derivation of Transformation Matrix

Performing transformation of in-plane stress components from local co-ordinate system to global co-ordinate system,

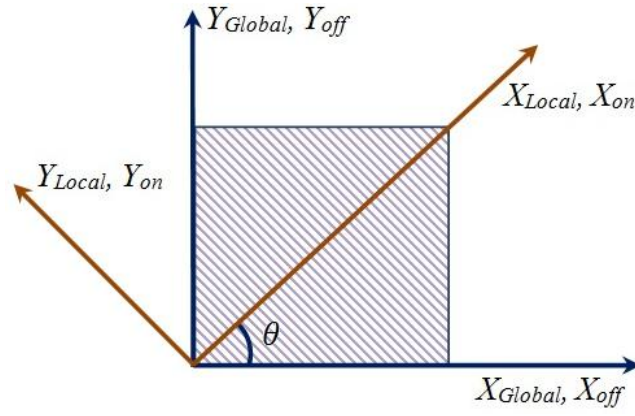


Fig. 6.5 Axis systems in a unidirectional lamina

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2 - n^2 \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \quad (6.33)$$

$$\{\sigma\}_{x,y} = [T]' \{\sigma\}_{1,2} \quad (6.34)$$

As transverse shear deformation is considered in the present study, thus performing transformation of out-of plane stress components from local co-ordinate system to global co-ordinate system,

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} m & -n \\ n & m \end{bmatrix} \begin{Bmatrix} \tau_4 \\ \tau_5 \end{Bmatrix} \quad (6.35)$$

$$\{\tau\}_{x,y} = [T] \{\tau\}_{1,2} \quad (6.36)$$

and $\cos \theta = m$, $\sin \theta = n$

In equation 6.34, $[T]'$ is termed as transformation matrix which is used to transfer in plane stress and strain components from local to global co-ordinate system and in 6.36 $[T]$ used to transfer out-of plane stress and strain components from local to global co-ordinate system.

6.9.1.3 Transformation of strain vector from Local to Global Co-Ordinate System

Transformation matrix derived in equation 6.34 can also be used to transform in plane strain components from local to global co-ordinate system.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{\varepsilon_{xy}}{2} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2 - n^2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{\varepsilon_6}{2} \end{Bmatrix} \quad (6.37)$$

$$\{\varepsilon\}_{x,y} = [T]' \{\varepsilon\}_{1,2} \quad (6.38)$$

Performing transformation of out-of plane stress components from local co-ordinate system to global co-ordinate system,

$$\begin{Bmatrix} \varepsilon_{yz} \\ \varepsilon_{xz} \end{Bmatrix} = \begin{bmatrix} m & -n \\ n & m \end{bmatrix} \begin{Bmatrix} \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix} \quad (6.39)$$

$$\{\varepsilon\}_{x,y} = [T] \{\varepsilon\}_{1,2} \quad (6.40)$$

6.8.1.4 Transformation of stress-strain matrix from Local to Global Co-Ordinate System

Combining equation 6.32, 6.33 and 6.37 we get in plane stress-strain relationship in global co-ordinate system,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2 - n^2 \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{\varepsilon_6}{2} \end{Bmatrix} \quad (6.41)$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} \quad (6.42)$$

And combining equation 6.31, 6.35 and 6.39 we get out-of plane stress-strain relationship in global co-ordinate system,

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} m & -n \\ n & m \end{bmatrix} \begin{bmatrix} C_{44} & 0 \\ 0 & C_{55} \end{bmatrix} \begin{bmatrix} m & n \\ -n & m \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad (6.43)$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} m^2 G_{23} - n^2 G_{31} & mn(G_{23} - G_{31}) \\ mn(G_{23} - G_{31}) & n^2 G_{23} + m^2 G_{31} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad (6.44)$$

$$\{\tau\} = [G_{ij}] \{\gamma\}_{1,2}$$

$[Q]_{x,y}$ in equation 6.42 represents the reduced stiffness of an individual lamina in global coordinate system. Elements of $[Q]_{x,y}$ has a specific significance. The elements shows the coupling effects of normal and shearing strain on normal stress and in the same way it also represents the coupling of shear strain and normal strain on shear stress. The terms of $[Q]_{x,y}$ are described below,

$$Q_{xx} = Q_{11} m^4 + 2(Q_{12} + Q_{66}) m^2 n^2 + Q_{22} n^4$$

$$Q_{xy} = (Q_{11} + Q_{22} - 4Q_{66}) m^2 n^2 + Q_{12}(m^4 + n^4)$$

$$Q_{xs} = (Q_{11} - Q_{22} - 2Q_{66}) nm^3 + (Q_{12} - Q_{22} + 2Q_{66}) mn^3$$

$$Q_{yy} = Q_{11} n^4 + 2(Q_{12} + 2Q_{66}) m^2 n^2 + Q_{22} m^4$$

$$Q_{ys} = (C_{11} - C_{12} - 2C_{66}) n^3 m + (C_{12} - C_{22} + 2C_{66}) m^3 n$$

$$Q_{ss} = (Q_{11} + Q_{22} - 2Q_{12}) m^2 n^2 + Q_{66} (m^2 - n^2)^2$$

6.8.2 Stress-Strain Relationship for Laminate

A laminate consist of series of lamina and bonded together to act as an integral structural element. A laminate is a layered construction shaped in the form of shell in our present study. To analyse a laminate following assumptions are considered in the present study.

Assumptions to analyse a laminate are-

- Each lamina of a laminate is quasi-homogeneous and anisotropic.
- Displacements are continuous throughout the laminate.
- Deformations of the laminate are small.
- Strain-displacement and stress-strain relationship is linear.
- Transverse shear deformations are considered to be small as the laminate is also thin. The transverse plane of bending remains plane before and after bending but not necessary to be normal to the mid-plane after bending. To consider the effects of shear deformation, a shear correction factor '1' is assumed.
- The bond between plies in a laminate are perfect and no slipping will occur between plies.

Combining equation 6.12 and 6.42 we get in plane stress-strain relationship for a laminate,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} \left\{ \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \right\}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (6.45)$$

From equation 6.44 we get out-off plane stress-strain relationship for a laminate,

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} m^2 G_{23} - n^2 G_{31} & mn(G_{23} - G_{31}) \\ mn(G_{23} - G_{31}) & n^2 G_{23} + m^2 G_{31} \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} \quad \because \kappa_{xz} = \kappa_{yz} = 0 \quad (6.46)$$

6.8.3 Equilibrium Equations

Equation 6.42 and 6.44 represents in plane and out-off plane stress components of the 'k'th lamina as shown in Fig. 6.6(b). To get the force and moment resultants of a lamina it is required to integrate the stresses over its thickness (h) and they are shown in Fig. 6.6(a).

Stress varies discontinuously along the fibers and to get the stress resultants of a laminate it is requires adding the stress resultant for each layer numerically.

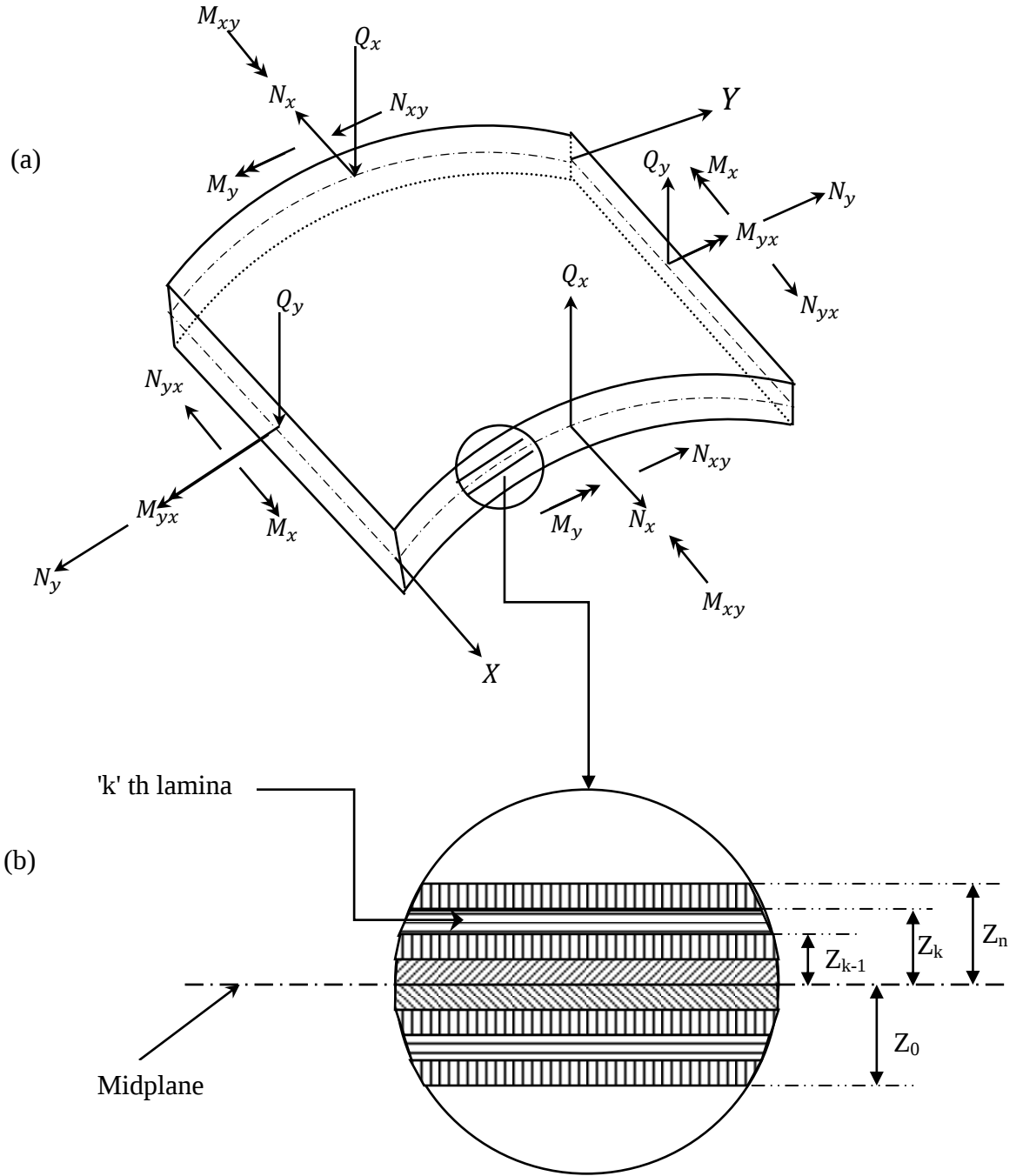


Fig. 6.6 (a) Generalized force and moment resultants; **(b)** laminate at its k th lamina

$$\{F\} = \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \sigma_x Z \\ \sigma_y Z \\ \tau_{xy} Z \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \quad (6.47)$$

In that way the normal stress resultants of a laminate are,

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \left\{ \sum_{k=1}^n \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \int_{z_{k-1}}^{z_k} dz + \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \int_{z_{k-1}}^{z_k} z dz \right\} \quad (6.48)$$

Bending stress resultants are,

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \left\{ \sum_{k=1}^n \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \int_{z_{k-1}}^{z_k} z dz + \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \int_{z_{k-1}}^{z_k} z^2 dz \right\} \quad (6.49)$$

Transverse shear resultants are,

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \left\{ \sum_{k=1}^n \kappa \begin{bmatrix} m^2 G_{23} - n^2 G_{31} & mn(G_{23} - G_{31}) \\ mn(G_{23} - G_{31}) & n^2 G_{23} + m^2 G_{31} \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} \int_{z_{k-1}}^{z_k} z dz \right\} \quad (6.50)$$

Where σ_x and σ_y are the normal stresses along X and Y directions, respectively and τ_{yz} , τ_{xz} and τ_{xy} are shear stresses in XY, XZ and YZ planes, respectively. The thickness of a laminate is denoted by h. Here ' κ ' is the shear correction factor to consider the transverse shear stress and for the present case it is considered '1' as the thickness is very small. In the present case upward deflection, tensile inplane forces, hogging moments and inplane shears tending to rotate the shell element anti-clockwise are taken positive.

Equations (6.48), (6.49) and (6.50) are combined to obtain

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} & 0 & 0 \\ A_{12} & A_{22} & A_{23} & B_{12} & B_{22} & B_{23} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} & 0 & 0 \\ B_{12} & B_{22} & B_{23} & D_{12} & D_{22} & D_{23} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{11} & S_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{12} & S_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \gamma_{yz}^0 \\ \gamma_{zx}^0 \end{Bmatrix} \quad (6.51)$$

$$\{F\} = [D] \{\varepsilon\} \quad (6.52)$$

Where,

$$A_{ij} = \sum_{k=1}^n (Q_{ij})_k (z_k - z_{k-1}) \quad B_{ij} = \frac{1}{2} \sum_{k=1}^n (Q_{ij})_k (z_k^2 - z_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^n (Q_{ij})_k (z_k^3 - z_{k-1}^3) \quad S_{ij} = \sum_{k=1}^n (G_{ij})_k (z_k - z_{k-1})$$

In the above equations z_k and z_{k-1} are the distances measured from the mid-plane of a laminate to the bottom of the k th and $(k-1)$ th laminate, respectively. $(Q_{ij})_k$ is the in plane elastic constant matrix for the k th lamina in global co-ordinate system and $(G_{ij})_k$ is the out-of plane elastic constant matrix for the k th lamina in global co-ordinate system. The 8×8 matrix in equation (6.51) is termed as the laminate stiffness matrix [D].

6.9 Governing Equations of Shell Bending

The governing equation of a composite shell is derived based on the principle of minimum total potential energy where the total potential energy ' π ' is expressed as sum of strain energy ' U ' and work done due to external load ' V '.

$$\pi = U + V \quad (6.53)$$

Strain energy of the shell is expressed as,

$$U = \frac{1}{2} \int_V \{\sigma\} \{\varepsilon\} dv \quad (6.54)$$

And work done by external load,

$$V = \iint \{u\}^T \{q\} dA \quad (6.55)$$

where ' v ' represents shell volume and „A“ shell area and $\{q\} = \{q_x \quad q_y \quad q_z \quad \mu_x \quad \mu_y\}^T$. In which q_x , q_y and q_z are the uniformly distributed loads per unit area along X, Y and Z axes respectively, and μ_x and μ_y are the moments per unit area along X and Y axes, respectively. In case of shells subjected to transverse loads only, q_x , q_y , μ_x and μ_y vanish and q_z remains.

6.10 Element Stiffness Matrix

The strain energy of the shell is expressed using finite element formulations as expressed before,

$$U_1 = \frac{1}{2} \iint_A \{d_e\}^T [B]^T [D] [B] \{d_e\} dx dy \quad (6.56)$$

And the work done is expressed as

$$V = - \iint_A \{d_e\}^T [N]^T [q] dx dy \quad (6.57)$$

To minimize the total potential energy of the shell with respect to its deformations, the shell has to satisfy,

$$\frac{\partial \pi}{\partial \{d_e\}} = 0 \quad (6.58)$$

Using equations 6.56 and 6.57 we get from equation 6.58

$$\left(\iint_A [B]^T [D] [B] dx dy \right) \{d_e\} - \left(\iint_A [N]^T [q] dx dy \right) = 0 \quad (6.59)$$

$$[K_e] \{d_e\} = \{Q_e\} \quad (6.60)$$

$$[K_e] = \iint_A [B]^T [D] [B] dx dy, \quad \{Q_e\} = \iint_A [N]^T [q] dx dy \quad (6.61)$$

The two-dimensional integral is converted to isoparametric coordinates and is carried out by 2×2 Gauss quadrature because the shape functions are derived from a cubic interpolation polynomial and it is an established fact that a polynomial of degree 2n-1 is integrated exactly by n point Gauss quadrature. The two-dimensional integral is converted to isoparametric coordinates and is carried out by 2×2 Gauss quadrature because the shape functions are derived from a cubic interpolation polynomial and it is an established fact that a polynomial of degree 2n-1 is integrated exactly by n point Gauss quadrature.

6.11 Element Load Vector

The consistent load vector $\{P_e\}$ is given by,

$$\{P_e\} = \iint_A [N]^T \{q\} dx dy \quad (6.62)$$

The area integral is evaluated by 2x2 Gauss quadrature like the stiffness matrix. And $\{q\} = \{q_x \ q_y \ q_z \ \mu_x \ \mu_y\}^T$. In which q_x , q_y and q_z are the uniformly distributed loads per unit area along X, Y and Z axes respectively, and μ_x and μ_y are the moments per unit area along X and Y axes, respectively. In case of shells subjected to transverse loads only, q_x , q_y , μ_x and μ_y vanish and q_z remains.

6.12 Failure Criteria

Independent Failure Criteria

Maximum Stress Criteria: According to maximum stress theory, the failure initiates if at least one of the criteria is satisfied,

$$\frac{\sigma_1}{F_x} \geq 1, \quad \frac{\sigma_2}{F_y} \geq 1, \quad \frac{\tau_{12}}{F_s} \geq 1 \quad (6.63)$$

The theory predicts the failure of the laminate based on the uniaxial test data. It does not consider the effect of biaxial stresses. It considers only the ultimate strength of the laminate in particular direction for the particular type of failure. There is no interaction between the stresses. In this way, the strength of the ply for multiaxial load states is over-predicted. However it is extensively used in the industry because of their simplicity.

Maximum Strain Criteria: According to maximum strain criteria failure occurs if any of the following is reached,

$$\frac{\epsilon_1}{\epsilon_1^u} \geq 1, \quad \frac{\epsilon_2}{\epsilon_2^u} \geq 1, \quad \frac{\epsilon_{12}}{\epsilon_{12}^u} \geq 1 \quad (6.64)$$

Like maximum stress theory, this theory also predicts the failure of the laminate based on the uniaxial test data. This theory also does not consider the effect of biaxial stresses. It considers only the ultimate strain of the laminate in particular direction for the particular type of failure. There is no interaction between strains, but interaction exists between the stress components caused by the Poisson's ratio effect.

Interactive Failure Criteria

Failure can be predicted by the following tensorial form,

$$F_i \sigma_i + F_{ij} \sigma_i \sigma_j + F_{ijk} \sigma_i \sigma_j \sigma_k + \dots \geq 1 \quad (6.65)$$

which when expanded in two dimensional form gives,

$$F_1 \sigma_1 + F_2 \sigma_2 + F_{11} \sigma_1^2 + F_{22} \sigma_2^2 + F_{66} \tau_{12}^2 + 2F_{12} \sigma_1 \sigma_2 \geq 1 \quad (6.66)$$

The failure indices for different theories are enlisted in Table 6.2.

Tsai-Hill criteria: This is one example of many criteria which attempt to take account of interactions in a multi-axial state of stress. It is basically based on Von Mises yield criterion. As it happens for the Von Mises criterion, different tensile and compression behaviors of the material are not accounted for. This can result in underestimation of the maximum loads that can be applied when compared to other failure theory. Thus, this criterion is not appropriate for materials with different tensile and compression behaviors.

Tsai-Wu criteria: This is again a quadratic failure criterion, which is most extensively used in the design of Composites. The linear stress terms in the above expression are responsible for possible differences in tensile and compressive strengths. The quadratic stress terms describe an ellipsoid in stress space. The failure criterion predicts the onset of failure but not the failure modes. The most inconvenient aspect of the failure criterion is the determination of the interaction term F_{12} , because, biaxial tests are required to determine the F_{12} . The theory does not consider the effect of initiating failure mechanisms. However it does predict the final failure quite in agreement with the experimental results. Being linear-elastic it could not predict the large non-linear strains observed in cases where high lamina shear is involved.

Hoffmann criteria: Basically the Hoffman criterion is a modification of the criterion proposed by Hill through inclusion of terms that are linear in the stress. Thus the 'no difference' between the tensile and compressive strength is obviated.

Table 6.2 Failure Indices for Different Interactive Failure Criteria

Failure Indices	Tsai Hill ^(a)	Tsai Wu	Hoffmann
F_1	0	$\left(\frac{1}{F_{XT}} - \frac{1}{F_{XC}}\right)$	$\left(\frac{1}{F_{XT}} - \frac{1}{F_{XC}}\right)$
F_2	0	$\left(\frac{1}{F_{YT}} - \frac{1}{F_{YC}}\right)$	$\left(\frac{1}{F_{YT}} - \frac{1}{F_{YC}}\right)$
F_{11}	$\frac{1}{F_X^2}$	$\frac{1}{F_{XT}F_{XC}}$	$\frac{1}{F_{XT}F_{XC}}$
F_{22}	$\frac{1}{F_Y^2}$	$\frac{1}{F_{YT}F_{YC}}$	$\frac{1}{F_{YT}F_{YC}}$
F_{66}	$\frac{1}{F_S^2}$	$\frac{1}{F_S^2}$	$\frac{1}{F_S^2}$
F_{12}	$-\frac{1}{2X^2}$	$-\frac{1}{2} \sqrt{F_{XT}F_{XC}F_{YT}F_{YC}}$	$-\frac{1}{2F_{XT}F_{XC}}$

If $\sigma_1 > 0$, $F_X = F_{XT}$ else $F_X = F_{XC}$. Similarly if $\sigma_2 > 0$, $F_Y = F_{YT}$ else $F_Y = F_{YC}$

Chapter 7

NUMERICAL STUDY

7.1 General

Using the mathematical formulation presented in the previous chapter, a number of numerical problems of laminated shells are analysed and results are presented in tabular and graphical forms.

7.2 Details of the Numerical Problems

The shell structure is divided into finite numbers of elements in the finite element method. These curved elements are connected at nodal points and by using the connected piece of elements which ultimately forms the mesh, the actual continuum is represented. It has been found that accuracy of the solution of this numerical technique is increased by using more numbers of elements in the mesh to represent the displacement field of the continuum or the shell structure. But using more numbers of elements results in more computational time consumption and moreover, if the element size becomes very small then also erroneous results are produced. To optimize the numbers of elements, convergence study of the mesh size is required. In order to serve the purpose, a convergence study through the static displacement for different mesh size is presented in Table 7.1 and the values are compared with the results obtained previously by Reddy (1984).

Table 7.1 Non-Dimensional central deflections ($w \times 10^3$) of simply supported composite spherical shell under uniformly distributed load

Lamination	0°/90°	0°/90°/0°	0°/90°/90°/0°
J. N. Reddy (1984)	16.980	6.697	6.833
Present FEM (2×2)	8.294	5.116	4.644
Present FEM (4×4)	16.898	6.724	6.820
Present FEM (6×6)	16.969	6.710	6.826
Present FEM (8×8)	17.009	6.707	6.835
Present FEM (10×10)	16.934	6.711	6.827

$$a/b = 1, a/h = 100, E_1 = 25 E_2, G_{12} = G_{13} = 0.5 E_{22}, G_{23} = 0.2 E_{22}, \nu = 0.25, E_2 = 10^6 \text{ N/cm}^2, R/a = 10^{30}$$

Table 7.2 Comparison of first ply failure loads in Newton for a $(0^\circ_2 / 90^\circ)_s$ plate

Failure Criteria	Thickness (mm)	First Ply Failure Load (Kam et al.)	First Ply Failure Load (Present Formulation)
Maximum Stress	105.26	108.26	112.14
Hoffmann		106.45	104.40
Tsai Wu		112.77	110.50
Tsai Hill		107.06	104.40

Note: Length = 100mm, ply thickness = 0.155mm, load details = central point load.

First ply failure loads evaluated using the present formulation are compared with the linear failure loads reported by Kam et al [1996] in Table 7.2 to establish the precision of the first ply failure formulation. The material properties of the shell for both the benchmark problem and the authors' own problems are presented in Table 7.3. For the authors' own problems geometric properties are presented in Table 7.4. The practical parametric variations include both angle and cross ply lamination of both anti-symmetric and symmetric stacking orders presented in Table 7.5. As for the boundary conditions, besides the conventional clamped (CCCC) and simply supported (SSSS) boundary conditions, two more edge conditions which are practically encountered are considered (CSSC and SCSC) as shown in Fig. 7.1(a). The Gauss point locations are shown in Fig. 7.1(b). Transverse uniformly distributed loading is considered.

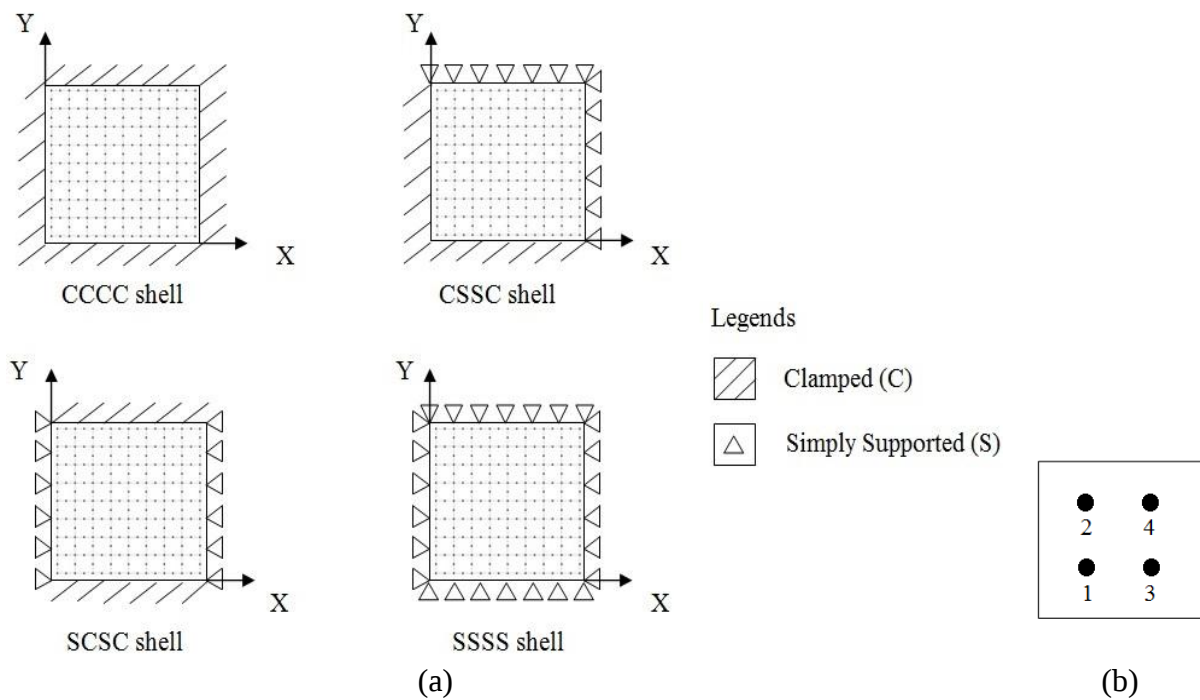
**Fig. 7.1(a)** Arrangement of boundary conditions; **(b)** Position of Gauss points.

Table 7.3 Material properties of graphite-epoxy

Material Constants	Values (GPa)	Strength Parameters	Values (MPa)	Strain Parameters	Values
E_{11}	142.50	F_{XT}	2193.5	ϵ_{XT}	0.01539
E_{22}	9.79	F_{XC}	2457	ϵ_{XC}	0.01724
E_{33}	9.79	$F_{YT} = F_{ZT}$	41.30	$\epsilon_{YT} = \epsilon_{ZT}$	0.00412
$G_{12} = G_{13}$	4.72	$F_{YC} = F_{ZC}$	206.8	$\epsilon_{YC} = \epsilon_{ZC}$	0.02112
G_{23}	1.192	$F_{s_{XY}}$	61.28	$\epsilon_{s_{XY}}$	0.05141
$\nu_{12} = \nu_{13}$	0.27	$F_{s_{YZ}}$	78.78	$\epsilon_{s_{YZ}}$	0.01669
ν_{23}	0.25	$F_{s_{XZ}}$	78.78	$\epsilon_{s_{XZ}}$	0.01669

Table 7.4 Geometric dimensions of the cylindrical shell

Shell Parameters	Values (mm)
Length (a)	1000
Width (b)	1000
Thickness (h)	10
Radius of Curvature (R)	750

Table 7.5 Different types of lamina

Lamina name	Stacking orders
Anti-symmetric Cross Ply (ASCP)	0°/90°
Anti-symmetric Angle Ply (ASAP)	45°/-45°
Symmetric Cross Ply (SYCP)	0°/90°/0°
Symmetric Angle Ply (SYAP)	45°/-45°/45°

It is observed that four failure criteria viz. Maximum stress criterion, Tsai-Hill criterion, Hoffman criterion and Tsai-Wu criterion have been used by most researchers the papers. In the present paper, the author considers these failure criteria as representative and failure loads are presented against each criterion in the tables (Table 7.6 to Table 7.9). Plies are numbered from the top to the bottom of the laminate i.e. the topmost ply is numbered one and bottommost ply has the highest ply number.

7.3 First Ply and Ultimate Ply Failure Load for Different Boundary Conditions

The First Ply Failure Load (FPFL) and the Ultimate Ply Failure Load (UPFL) of the composite cylindrical shells are presented in Table 7.6 to 7.9 for different boundary conditions. These boundary conditions are denoted by CCCC (all 4 edges are clamped),

CSSC (two adjacent edges clamped and simply supported), SCSC (the arch edges simply supported and beam edges clamped) and SSSS (all 4 edges simply supported) respectively. The positions of boundary constraints are explained in Fig. 7.1(a). There are other practical boundary conditions as well but those are not considered for the sake of brevity.

Table 7.6 contains the failure loads of the clamped shells obtained from different stress based failure criteria. As in practical civil engineering the FPFL corresponding to the maximum strain criterion is always more than the load corresponding to maximum permissible deflection, the maximum strain criterion is excluded in the present study. Of all the FPFL values corresponding to different failure criterion, the one which is least is accepted as the FPFL value. Table 7.6 reveals that different failure criteria become the guiding factor for different laminations. The ASCP ($0^\circ/90^\circ$) shell fails in Tsai Hill criterion, the ASAP ($45^\circ/-45^\circ$) and SYAP ($45^\circ/-45^\circ/45^\circ$) ones fail in Tsai Wu criterion while Hoffmann's failure criterion governs the failure of SYCP ($0^\circ/90^\circ/0^\circ$) shell. This indicates that in order to get FPFL value all the stress based criteria should be simultaneously be worked out to assess the acceptance value. The cross ply shells, for clamped boundary conditions, exhibit predominantly high values of FPFLs compared to angle ply ones. Among the cross ply shells again, the symmetric $0^\circ/90^\circ/0^\circ$ laminate shows the best performance.

It is also interesting to note that among two shell configurations the one which shows higher value of the FPFL may not correspond to the higher values of UPFL. In fact for clamped boundary conditions, the symmetric angle ply laminates exhibits the highest value of UPFL. Since the successive failure of the elements lead to redistribution of stresses within the shell mass, the failure progress which utilises the maximum material strain yields the highest value of UPFL. For all the laminations the UPFL value is by far greater than the FPFL ones.

In practical civil engineering, apart from strength, serviceability is also an important parameter in design. A permissible deflection of span/250 which is 4 mm here is considered in the present study. For clamped shells the deflections corresponding to the FPFL values are less than 4 mm and hence they are not mentioned in Table 7.6.

Fig. 7.2 to 7.5 shows the failure progress for the clamped shells. The failed elements are marked in each lamina for the first ply failure load, the loads at which the 2nd to 5th elements fail, 25%, 50%, 75% of UPFL and finally the UPFL itself.

For the cross ply shells, the failure initiates parallel to the beam direction near the support and even at UPF condition some of the laminae remain undamaged near the arch supports. For the angle ply, however, the failure initiates from the edges near the arch support or form a corner.

Table 7.7 and Fig. 7.6 to 7.9 present the failure load information and the nature of progressive failure for the CSSC composite cylindrical shells. For this boundary condition the cross ply

shells are again better than the angle ply ones in terms of FPFL values, within which again the SYCP ($0^\circ/90^\circ/0^\circ$) laminate yields the highest value of FPFL. The UPFL are however higher in case of angle ply shells compared to the cross ply ones though marginally. It is interesting to note that the ASAP and SYCP shells have the load corresponding to the 4mm deflection is less than the FPFL. If this is taken in consideration then the ASCP laminate becomes the most preferred one when FPF and serviceability are practically combined to assess the load where practical failure initiates. Apart from that, for this boundary condition also the UPFLs are far greater than the loads at which failure initiates the failure progress in this class of boundary condition is totally unsymmetrical and concentrated near one beam edge predominantly for all the laminations.

The failure load and the failure progress data of SCSC shell are furnished in Table 7.8 and Fig. 7.10 to 7.13. For this symmetric boundary condition the cross ply fibres are oriented along the beam and arch direction parallel to the principal curvatures of the shell. This is why the cross ply shells exhibit much higher values of the FPF loads compared to the angle ply ones. As one sees for the outer boundary conditions the UPFL is highest for the antisymmetric shells. Interestingly, the failure progress does not show any unified tendency for this class of shell. For the $0^\circ/90^\circ$ one, the elements near the beam edges are prone to failure, for the $0^\circ/90^\circ/0^\circ$ one, some element along the arch edges are also involved while for $45^\circ/-45^\circ$ shell the failure is all along the boundaries. And for $45^\circ/-45^\circ/45^\circ$ the elements near the arch boundaries soften more in number under overloading.

Simply supported cylindrical shells are symmetrically restrained and the failure load and failure progress data are furnished in Table 7.9 and Fig 7.14 to 7.17. In three out of four laminations taken up here, the failure load is governed by serviceability criteria and only for the ASCP ($0^\circ/90^\circ$) the Hoffman and the Tsai Wu criteria yield identical results to determine the FPFL which is in fact the highest value at which the failure initiates. The SYAP yields the highest value of UPF load and all the values are by far greater than the loads at which the failure initiates. For this boundary condition the failure progress is from all the corners and all sides and huge number of elements fails at UPF stage.

When the failure loads corresponding to all the boundary conditions and laminations taken up here are compared, it is found that the fully clamped shells yield the highest value of FPFL while the simply supported ones exhibit the lowest values. This is quite expected because the number of boundary constraints determines the overall rigidity of the structure and in that sense the clamped shell has the maximum degrees of freedom locked and hence is the stiffest choice. Interestingly, the CSSC and the SCSC have equal number of boundary conditions locked but the failure loads show a wide variation. The SCSC shells have the two beam edges clamped and have much more improved failure load values compared to the CSSC ones. This is because a cylindrical shell transfers the loads and moments along two plan directions

parallel to its principal curvatures and when the fibres too (like in cross ply shells) are oriented in these directions, the overall improvement of its behaviour is noteworthy.

Table 7.6 First ply failure load and ultimate ply failure load for boundary condition CCCC

Type of Lamina	Failure criteria	First Ply Failure			Failed Element	Ultimate Ply Failure Load (N/m ²)	Load for 4mm deflection where such load is less than FPFL (N/m ²)
		Failure Load (N/m ²)	Failed Ply	Failed Gauss Point			
ASCP (0°/90°)	Maximum Stress	10.3228	2	2	1	478.5981	
	Hoffmann	6.8577	2	1	9	505.8733	
	Tsai-Hill	5.7379	2	1	9	500.0279	
	Tsai-Wu	8.8575	1	1	49	500.5425	
ASAP (45°/-45°)	Maximum Stress	7.0195	2	3	41	507.4703	
	Hoffmann	1.6683	2	2	57	462.5667	
	Tsai-Hill	1.9823	2	2	57	499.9356	
	Tsai-Wu	1.5315	2	2	57	510.6731	
SYCP (0°/90°/0°)	Maximum Stress	10.1159	3	3	64	478.6895	
	Hoffmann	9.1577	3	2	16	505.8049	
	Tsai-Hill	9.2073	3	2	16	505.634	
	Tsai-Wu	9.1984	3	2	16	507.3306	
SYAP (45°/-45°/45°)	Maximum Stress	6.8371	3	1	41	503.0381	
	Hoffmann	3.1147	3	1	64	507.8418	
	Tsai-Hill	4.0145	3	1	64	510.7128	
	Tsai-Wu	2.8143	3	1	64	509.8443	

Note: The numerical in *bold* signifies the *least failure load* obtained from the four failure criteria.

Table 7.7 First ply failure load and ultimate ply failure load for boundary condition CSSC

Type of Lamina	Failure criteria	First Ply Failure			Failed Element	Ultimate Ply Failure Load (N/m ²)	Load for 4mm deflection where such load is less than FPFL (N/m ²)
		Failure Load (N/m ²)	Failed Ply	Failed Gauss Point			
ASCP (0°/90°)	Maximum Stress	0.35	2	4	64	495.299	
	Hoffmann	0.3471	2	4	64	503.51	
	Tsai-Hill	0.35	2	4	64	496.1778	
	Tsai-Wu	0.3471	2	4	64	509.0279	
ASAP (45°/-45°)	Maximum Stress	3.1225	2	3	32	500.6833	0.186
	Hoffmann	0.2689	2	4	8	508.9233	
	Tsai-Hill	0.2951	2	4	8	493.7874	
	Tsai-Wu	0.2536	2	4	8	487.578	
SYCP (0°/90°/0°)	Maximum Stress	0.3918	3	4	64	504.185	0.1842
	Hoffmann	0.3869	3	4	64	476.519	
	Tsai-Hill	0.3917	3	4	64	502.3969	
	Tsai-Wu	0.3869	3	4	64	505.8702	
SYAP (45°/-45°/45°)	Maximum Stress	3.6842	3	1	48	497.9127	
	Hoffmann	0.3345	3	4	64	494.092	
	Tsai-Hill	0.3818	3	2	64	506.9131	
	Tsai-Wu	0.3085	3	4	64	510.7288	

Note: The numerical in *bold* signifies the *least failure load* obtained from the four failure criteria.

Table 7.8 First ply failure load and ultimate ply failure load for boundary condition SCSC

Type of Lamina	Failure criteria	First Ply Failure			Failed Element	Ultimate Ply Failure Load (N/m ²)	Load for 4mm deflection where such load is less than FPFL (N/m ²)
		Failure Load (N/m ²)	Failed Ply	Failed Gauss Point			
ASCP (0°/90°)	Maximum Stress	7.953	2	4	57	428.3582	
	Hoffmann	6.4961	2	1	9	508.5766	
	Tsai-Hill	5.4422	2	1	9	505.3152	
	Tsai-Wu	8.4558	2	4	57	507.1261	
ASAP (45°/-45°)	Maximum Stress	3.907	2	4	5	506.8386	
	Hoffmann	0.8236	2	1	8	507.7685	
	Tsai-Hill	0.9266	2	1	8	498.7938	
	Tsai-Wu	0.7697	2	1	8	509.5012	
SYCP (0°/90°/0°)	Maximum Stress	7.7739	3	2	7	401.7878	
	Hoffmann	9.0807	3	3	49	496.7425	
	Tsai-Hill	7.7009	3	2	7	509.901	
	Tsai-Wu	8.9952	3	2	7	506.9135	
SYAP (45°/-45°/45°)	Maximum Stress	4.2533	3	3	53	509.5024	
	Hoffmann	0.9201	3	4	64	507.4771	
	Tsai-Hill	1.1306	3	4	64	505.5793	
	Tsai-Wu	0.8375	3	4	64	507.9734	

Note: The numerical in *bold* signifies the *least failure load* obtained from the four failure criteria.

Table 7.9 First ply failure load and ultimate ply failure load for boundary condition SSSS

Type of Lamina	Failure criteria	First Ply Failure			Failed Element	Ultimate Ply Failure Load (N/m ²)	Load for 4mm deflection where such load is less than FPFL (N/m ²)
		Failure Load (N/m ²)	Failed Ply	Failed Gauss Point			
ASCP (0°/90°)	Maximum Stress	0.3395	2	4	64	509.9325	
	Hoffmann	0.3367	2	4	64	508.7325	
	Tsai-Hill	0.3394	2	4	64	427.1703	
	Tsai-Wu	0.3367	2	4	64	504.6303	
ASAP (45°/-45°)	Maximum Stress	2.5638	2	1	32	508.6448	0.0625
	Hoffmann	0.2643	2	3	57	490.1941	
	Tsai-Hill	0.3051	2	3	57	511.657	
	Tsai-Wu	0.2445	2	3	57	463.4713	
SYCP (0°/90°/0°)	Maximum Stress	0.3735	3	4	64	449.4924	0.087
	Hoffmann	0.3692	3	3	57	503.2839	
	Tsai-Hill	0.3733	3	3	57	441.8208	
	Tsai-Wu	0.3692	3	3	57	510.4998	
SYAP (45°/-45°/45°)	Maximum Stress	3.2116	3	3	40	482.8280	0.1428
	Hoffmann	0.3247	3	4	64	507.6206	
	Tsai-Hill	0.3598	3	3	1	511.1133	
	Tsai-Wu	0.2997	3	4	64	505.0427	

Note: The numerical in *bold* signifies the *least failure load* obtained from the four failure criteria.

7.4 Evaluation of Working Load and Values of Partial Safety Factor (PSF) and Load Factor (LF)

In the study carried out so far it is found that in a number of cases the FPFL values is more than the load corresponding to the 4 mm deflection. In these cases the load corresponding to the permissible deflection of 4 mm may be adopted as the working load and so the FPFL value may be divided with the load corresponding to the 4 mm deflection to get the Partial Safety Factor (PSF). In the rest of the cases the first ply failure occurs when the deflection is less than 4 mm which means that the material failure precedes the serviceability failure. Naturally in these cases the PSFs are unity. When the ultimate ply failure load is divided by the corresponding working load, the load factor is obtained. The partial safety factors and the load factors as explained here are furnished in Table 7.10 for the different laminations and boundary conditions. It is observed that the values of load factors are extremely high which means that the laminated composite have substantial amount of load resisting capacity beyond the working load and hence these materials are expected to exhibit a ductile failure. This property of laminated composite makes them suitable to be used in earthquake prone zones.

Table 7.10 The Partial Safety Factor (PSF) and the Load Factor (LF) values

Boundary Conditions	Lamination	PSF	LF
CCCC	ASCP (0°/90°)	1.0000	83.4100
	ASAP (45°/-45°)	1.0000	302.0351
	SYCP (0°/90°/0°)	1.0000	52.2718
	SYAP (45°/-45°/45°)	1.0000	178.7436
CSSC	ASCP (0°/90°)	1.0000	1426.9631
	ASAP (45°/-45°)	1.3630	2621.3860
	SYCP (0°/90°/0°)	2.1000	2586.9663
	SYAP (45°/-45°/45°)	1.0000	1601.5961
SCSC	ASCP (0°/90°)	1.0000	78.7105
	ASAP (45°/-45°)	1.0000	648.0366
	SYCP (0°/90°/0°)	1.0000	52.1741
	SYAP (45°/-45°/45°)	1.0000	603.6768
SSSS	ASCP (0°/90°)	1.0000	1268.6971
	ASAP (45°/-45°)	3.9120	7415.5408
	SYCP (0°/90°/0°)	4.2440	5078.4000
	SYAP (45°/-45°/45°)	2.0980	3381.1485

7.5 Failure Progress

It has already been discussed that the failure criterion through which the minimum value of the FPFL is obtained is often different from the criterion for which the UPFL has the least one. But the criterion from which the FPFL is obtained is more important than that of UPFL. This is why the failure progress according to this criterion is studied pictorially in discrete steps and lamina wise. The failed elements are coloured corresponding to first to fifth ply failure loads and corresponding to 0.25 UPFL, 0.5 UPFL, 0.75 UPFL and UPFL (see Fig. 7.2 to Fig. 7.17).

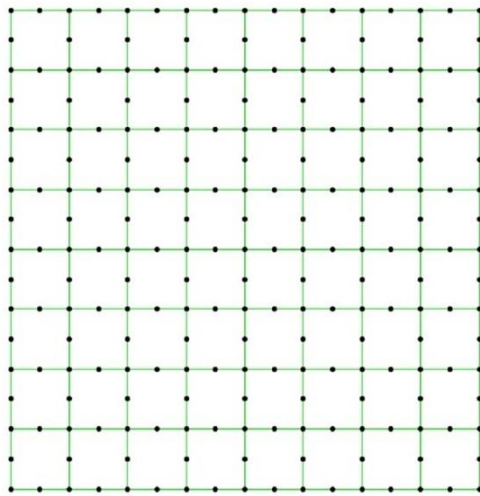
The location of the first ply failure point is extremely important to be known to a practicing engineer because any instrumentation needed for hidden flaw detection should start from that point. In other words if it is found that the point prone to first ply failure damage is free of any hidden flaw it can safely be concluded that the shell surface is free of any damage caused due to overloading.

Note:

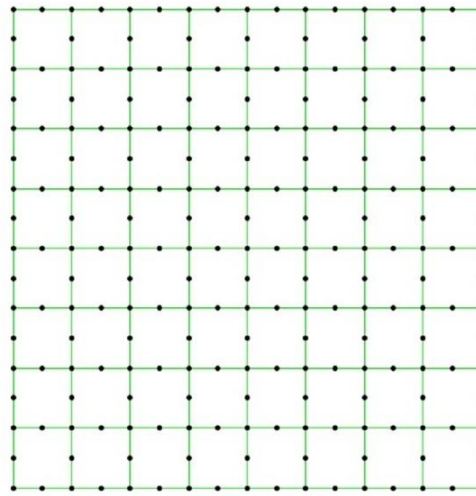
In all the following figures (Fig. 7.2 to Fig. 7.17),

- The coloured portions represent the “Failed Element”.
- The number written in coloured portion represents the “Number of Failure”.

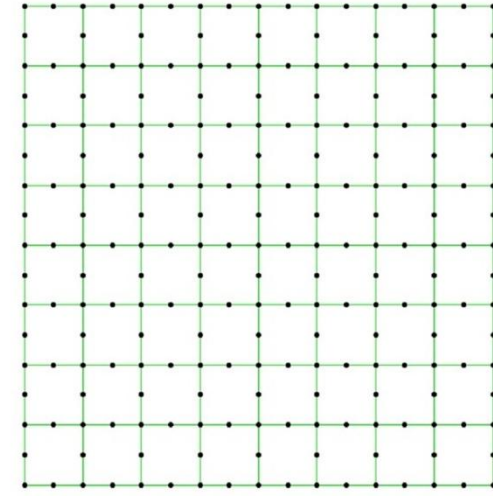
Fig. 7.2 Failure pattern of type: ASCP lamina for boundary condition: CCCC



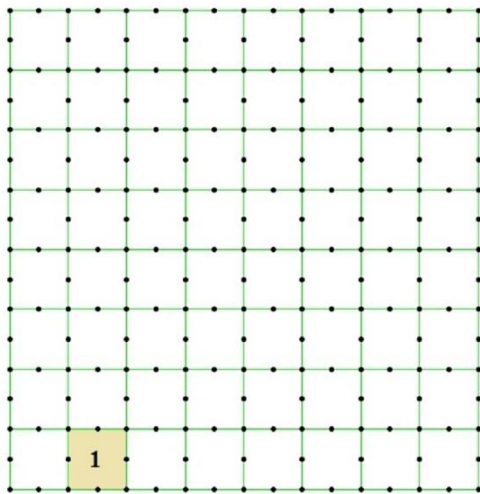
0° (FAILURE 1)



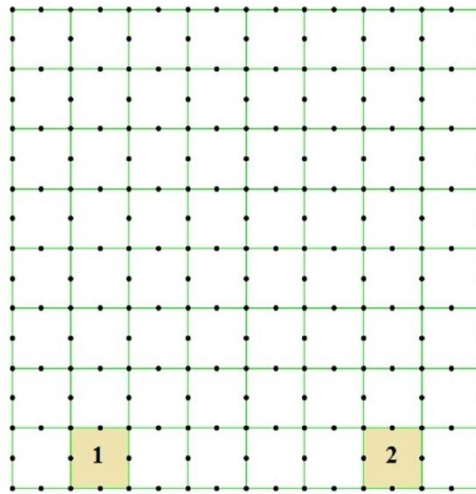
0° (FAILURE 2)



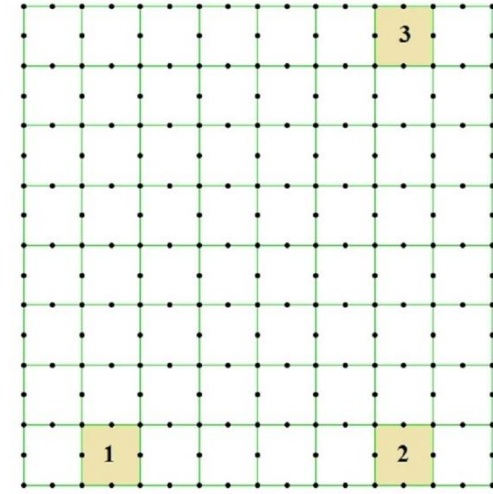
0° (FAILURE 3)



90° (FAILURE 1)

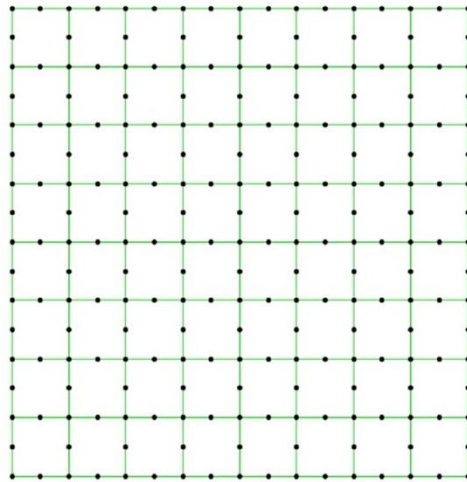


90° (FAILURE 2)

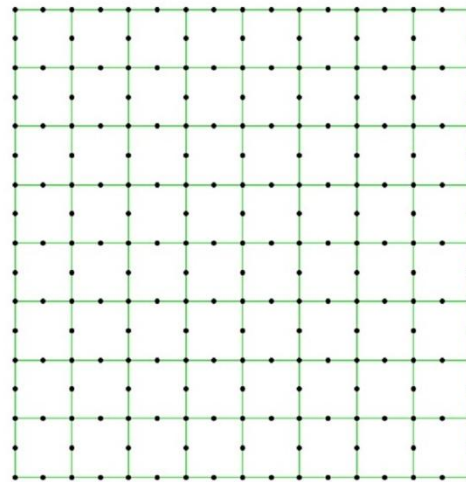


90° (FAILURE 3)

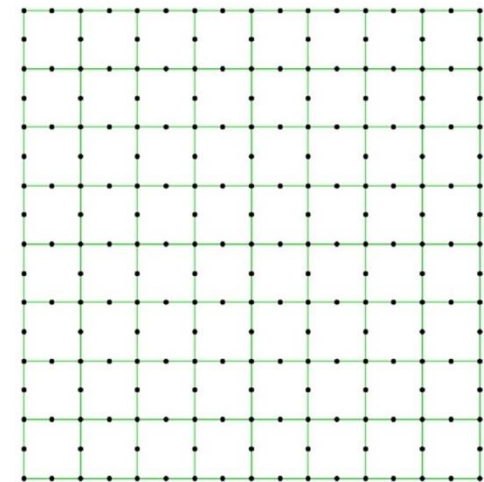
Fig. 7.2 Failure pattern of type: ASCP lamina for boundary condition: CCCC (contd.)



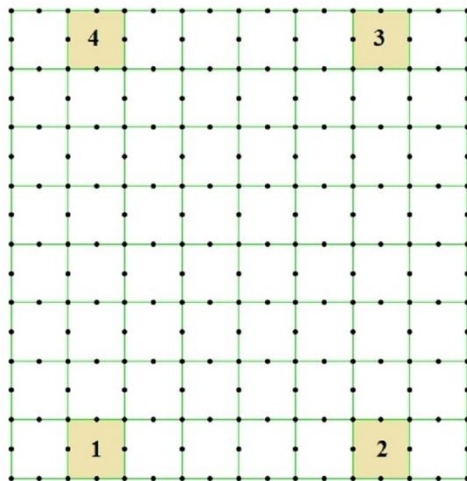
0° (FAILURE 4)



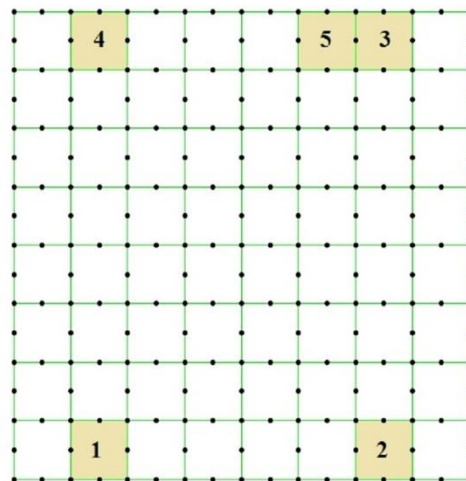
0° (FAILURE 5)



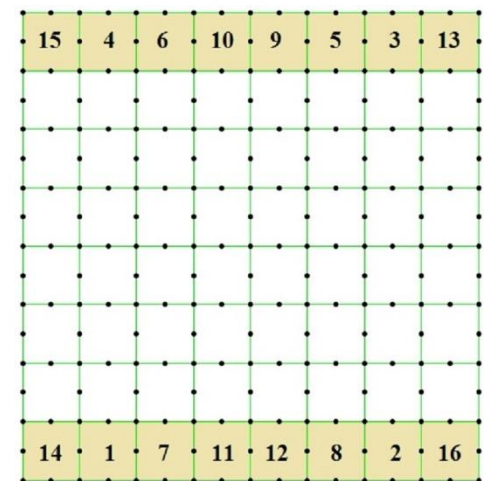
0° (0.25 UPF)



90° (FAILURE 4)

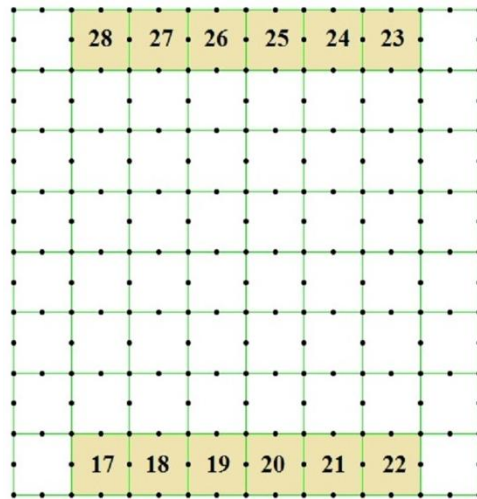


90° (FAILURE 5)

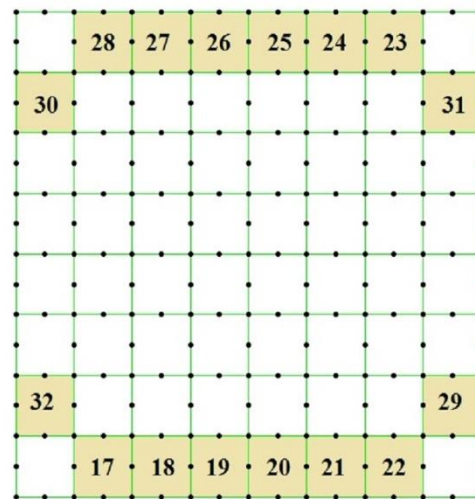


90° (0.25 UPF)

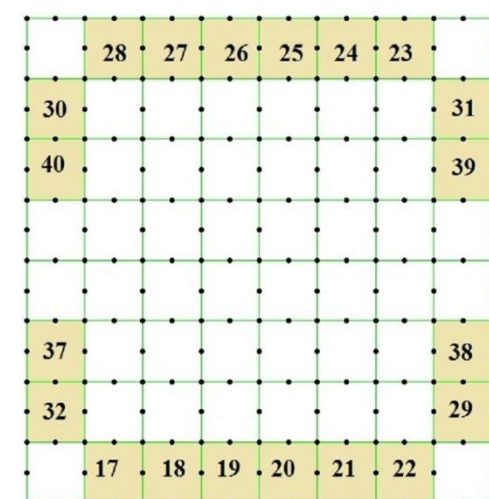
Fig. 7.2 Failure pattern of type: ASCP lamina for boundary condition: CCCC (contd.)



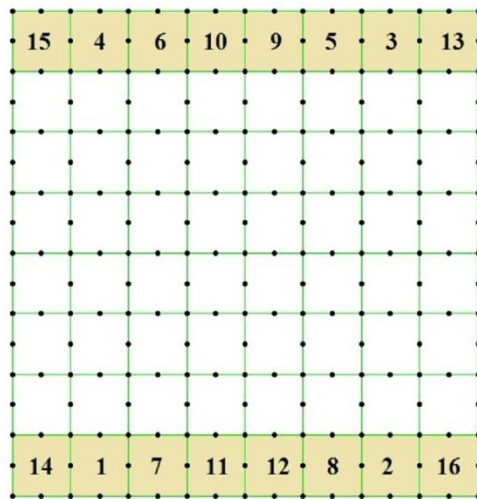
0° (0.5 UPF)



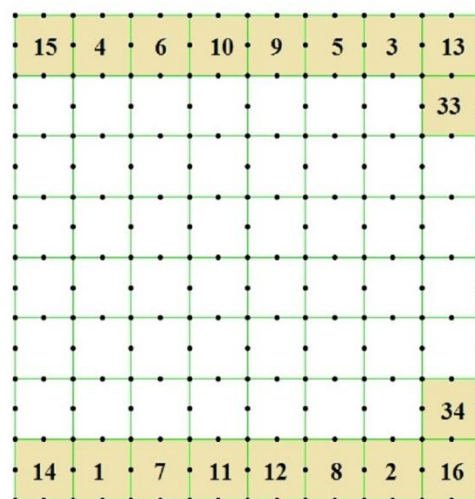
0° (0.75 UPF)



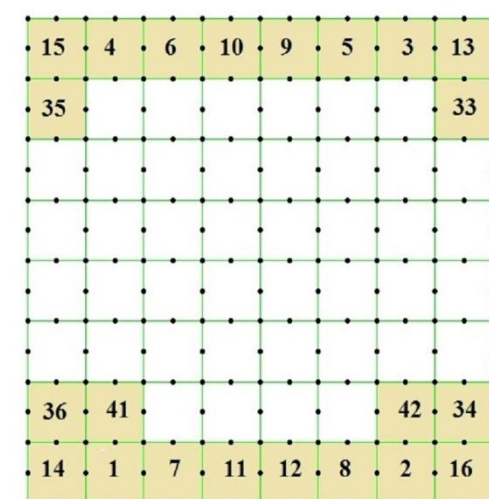
0° (UPF)



90° (0.5 UPF)

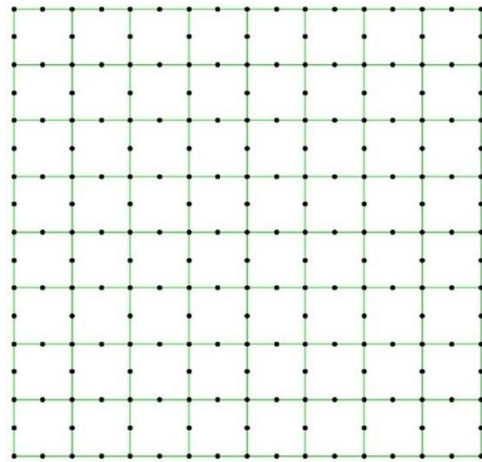


90° (0.75 UPF)

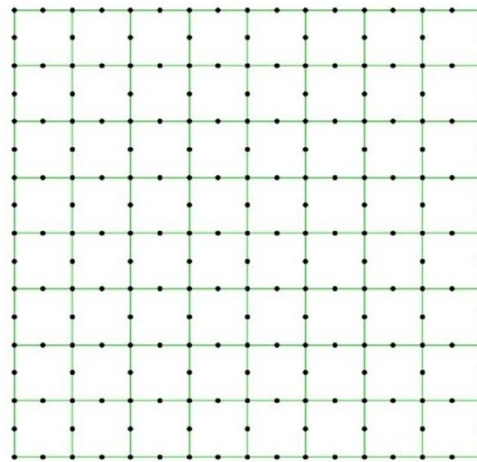


90° (UPF)

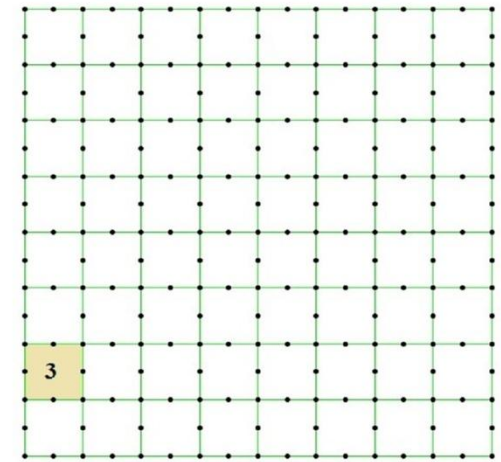
Fig. 7.3 Failure pattern of type: ASAP lamina for boundary condition: CCCC



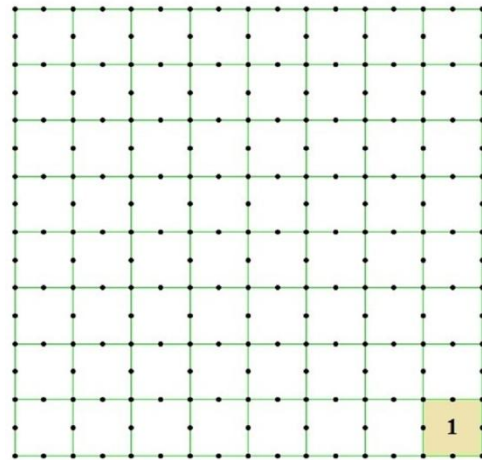
45° (FAILURE 1)



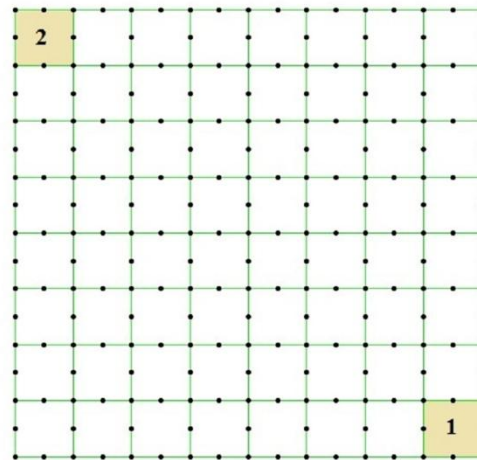
45° (FAILURE 2)



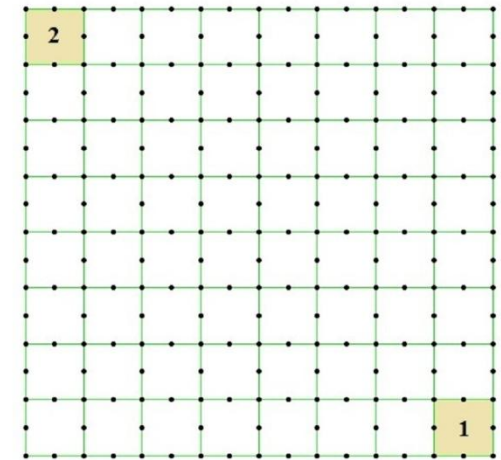
45° (FAILURE 3)



- 45° (FAILURE 1)

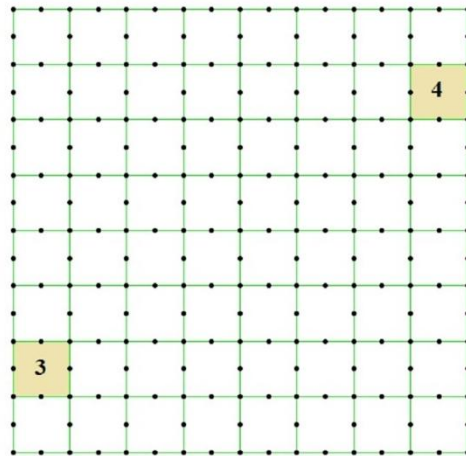


- 45° (FAILURE 2)

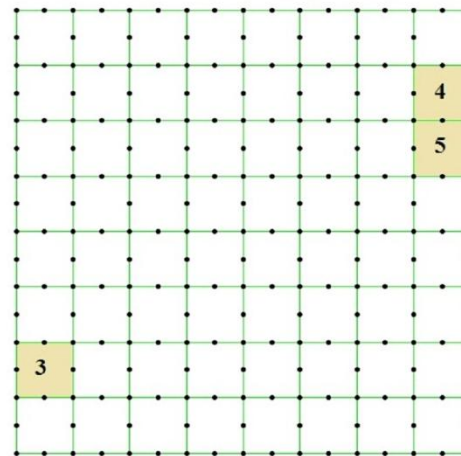


- 45° (FAILURE 3)

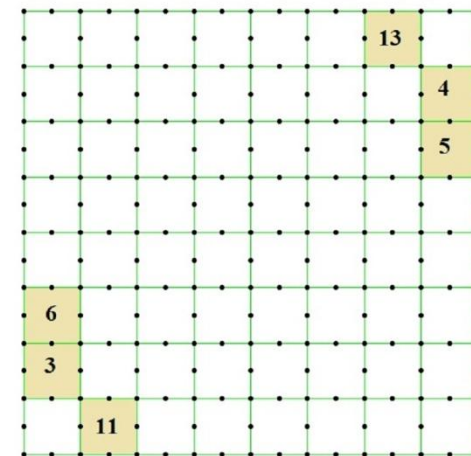
Fig. 7.3 Failure pattern of type: ASAP lamina for boundary condition: CCCC (contd.)



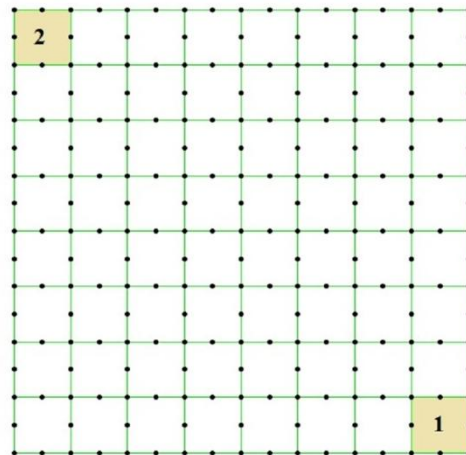
45° (FAILURE 4)



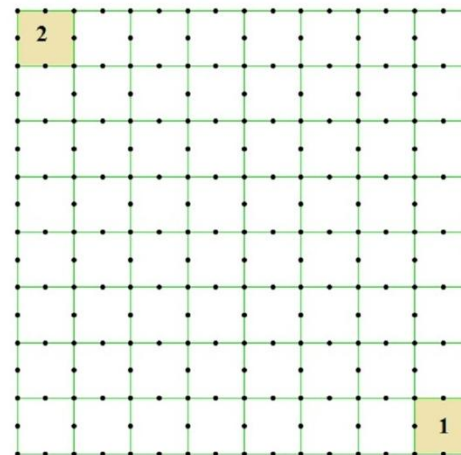
45° (FAILURE 5)



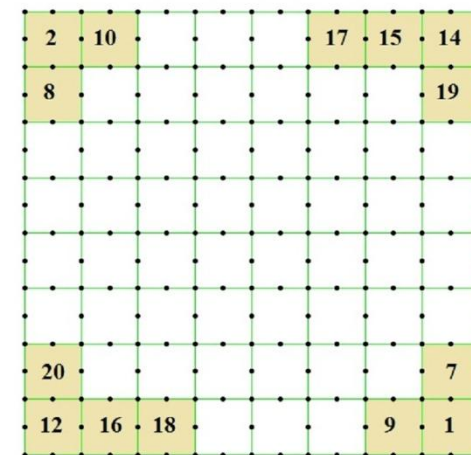
45° (0.25 UPF)



- 45° (FAILURE 4)

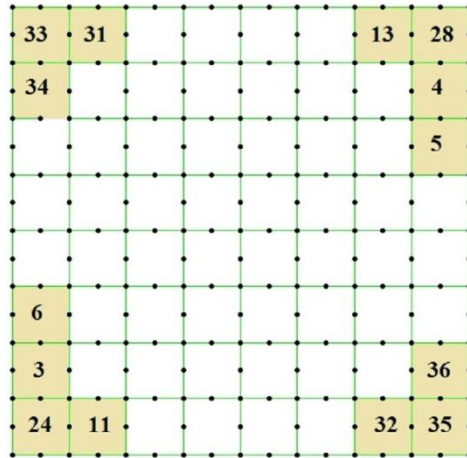


- 45° (FAILURE 5)

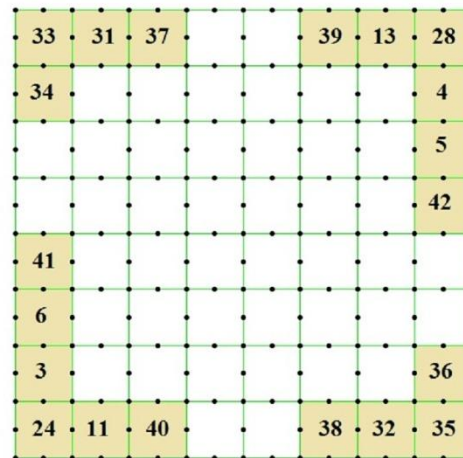


- 45° (0.25 UPF)

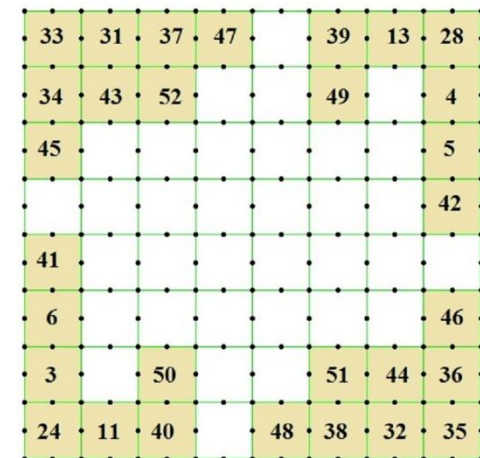
Fig. 7.3 Failure pattern of type: ASAP lamina for boundary condition: CCCC (contd.)



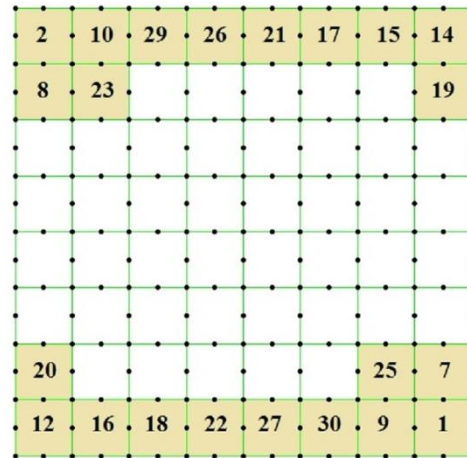
45° (0.5 UPF)



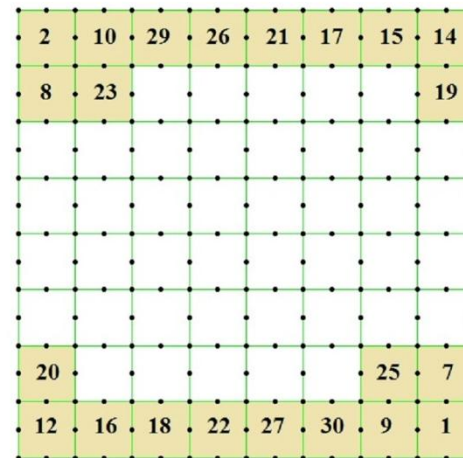
45° (0.75 UPF)



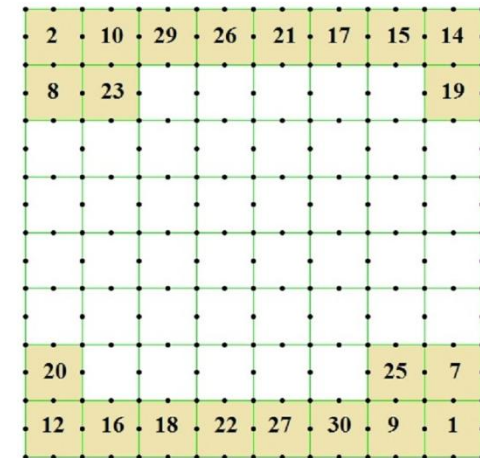
45° (UPF)



- 45° (0.5 UPF)

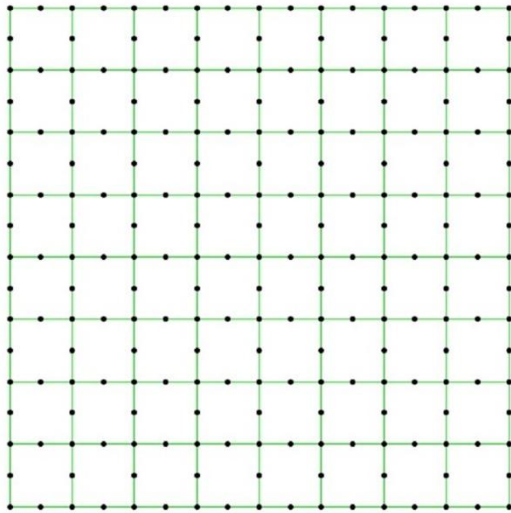


- 45° (0.75 UPF)

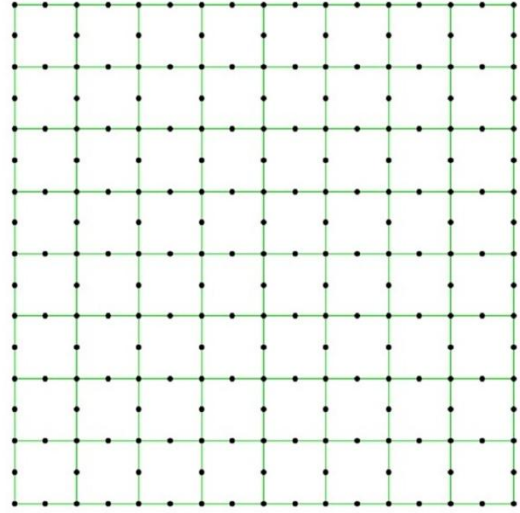


- 45° (UPF)

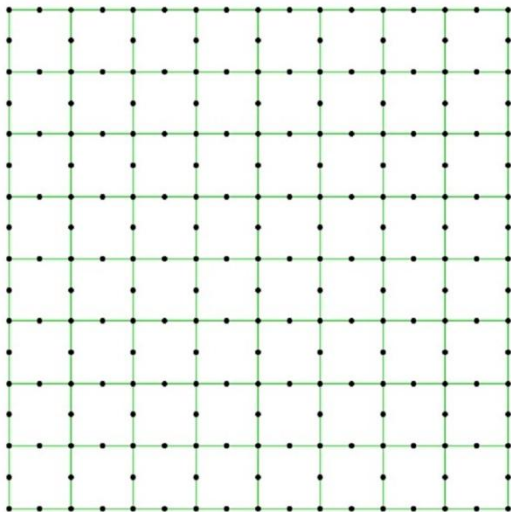
Fig. 7.4 Failure pattern of type: SYCP lamina for boundary condition: CCCC



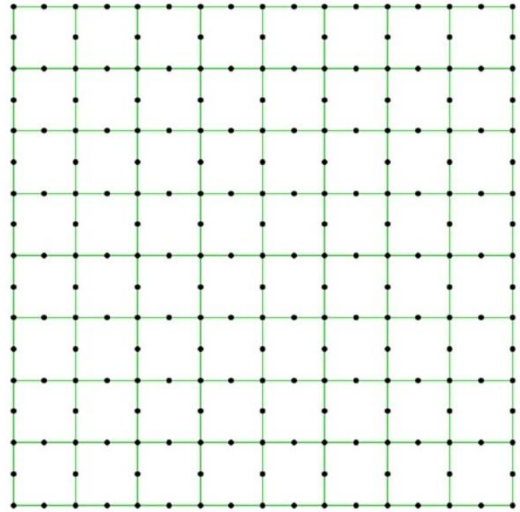
0° (FAILURE 1)



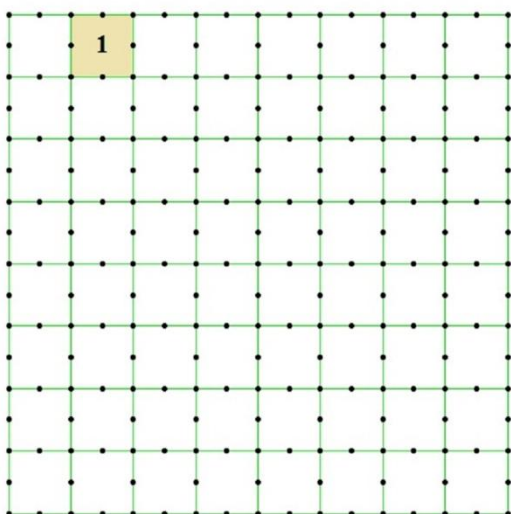
0° (FAILURE 2)



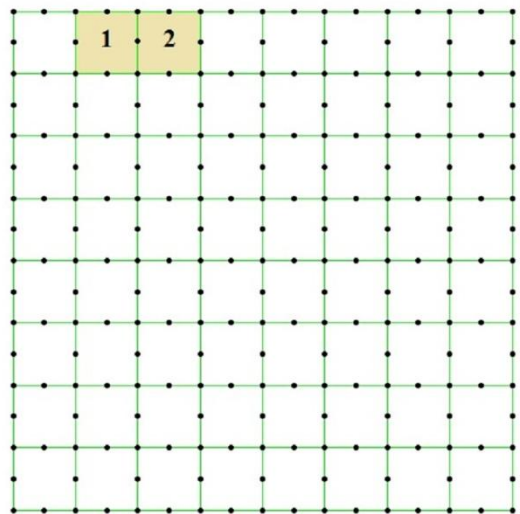
90° (FAILURE 1)



90° (FAILURE 2)

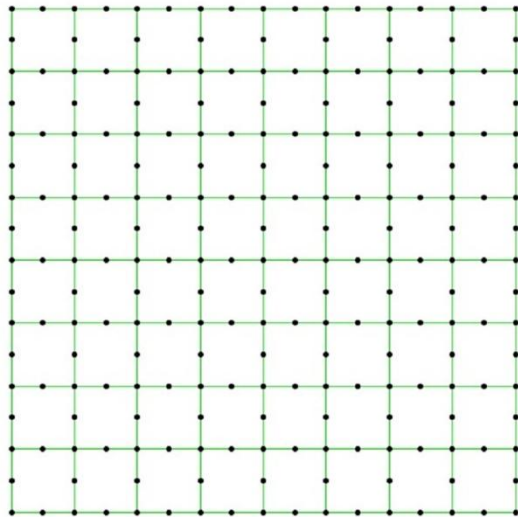


0° (FAILURE 1)

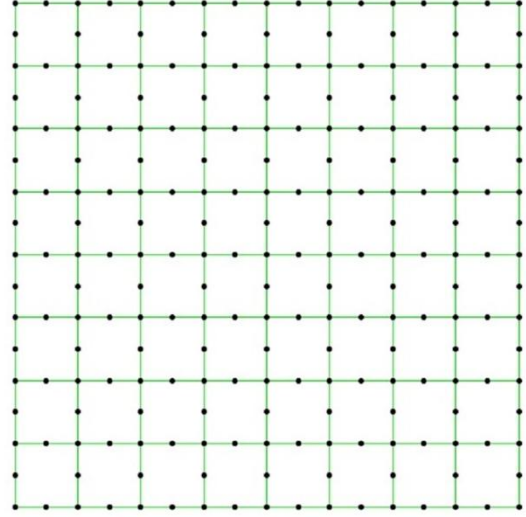


0° (FAILURE 2)

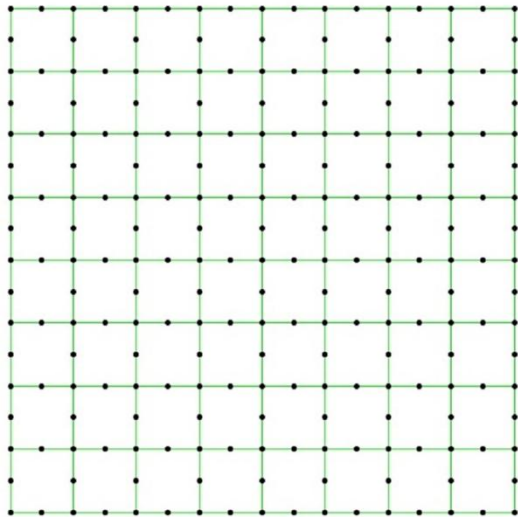
Fig. 7.4 Failure pattern of type: SYCP lamina for boundary condition: CCCC (contd.)



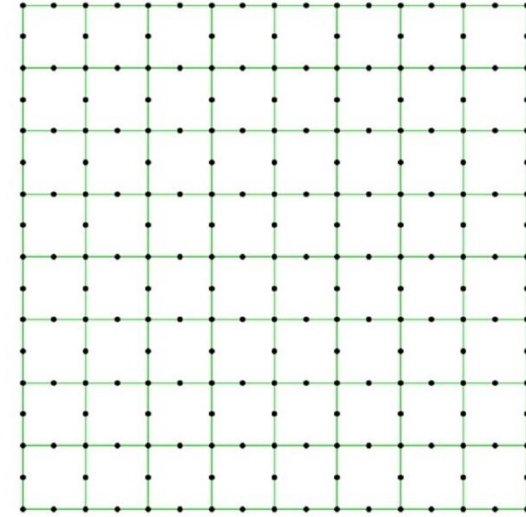
0° (FAILURE 3)



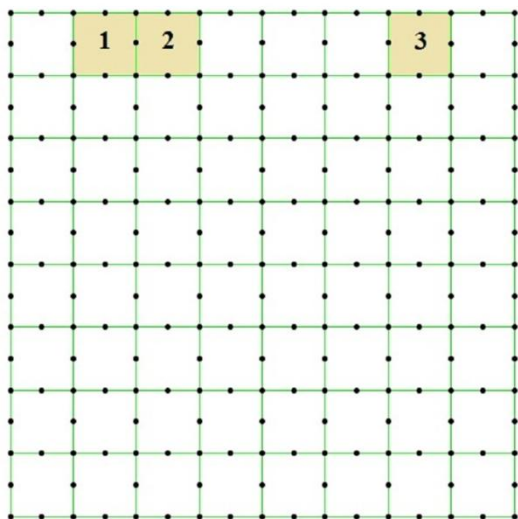
0° (FAILURE 4)



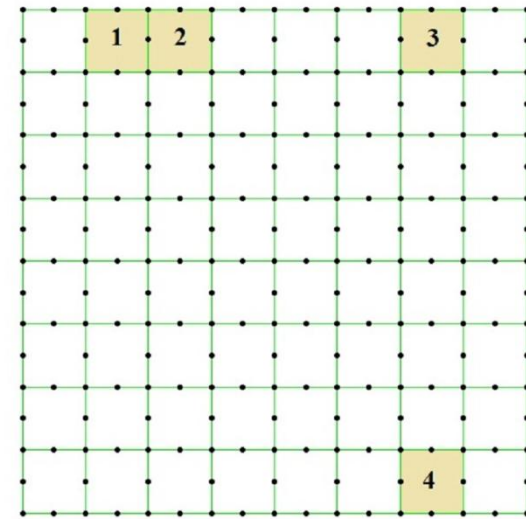
90° (FAILURE 3)



90° (FAILURE 4)

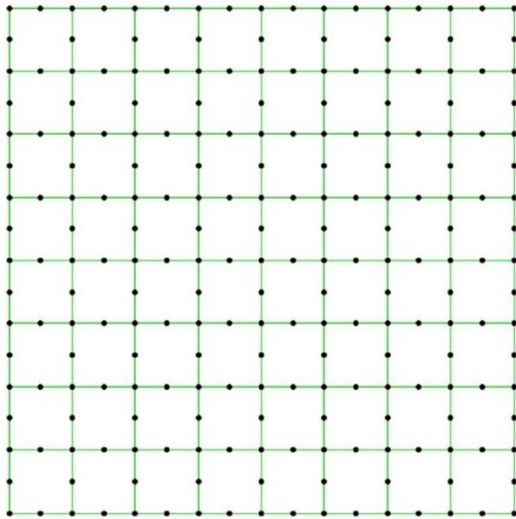


0° (FAILURE 3)

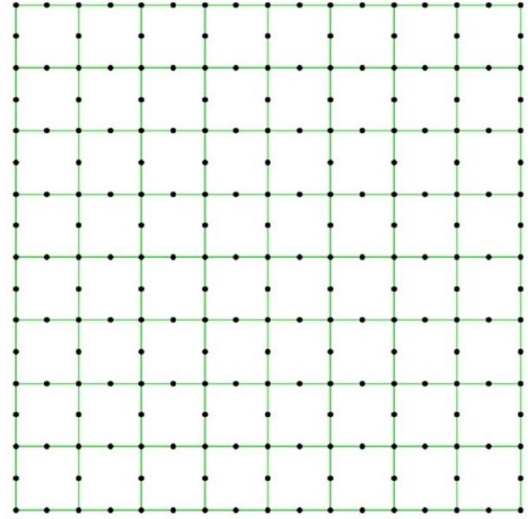


0° (FAILURE 4)

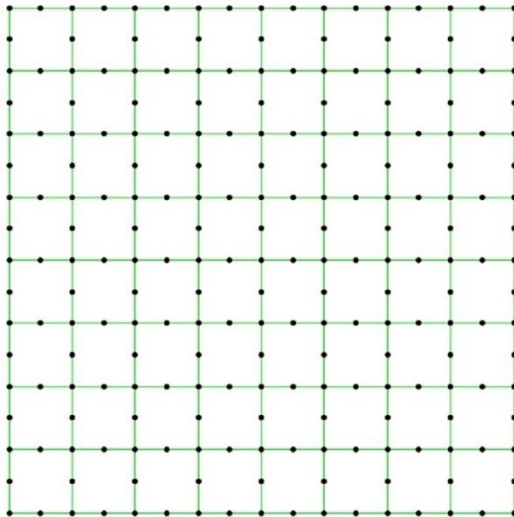
Fig. 7.4 Failure pattern of type: SYCP lamina for boundary condition: CCCC (contd.)



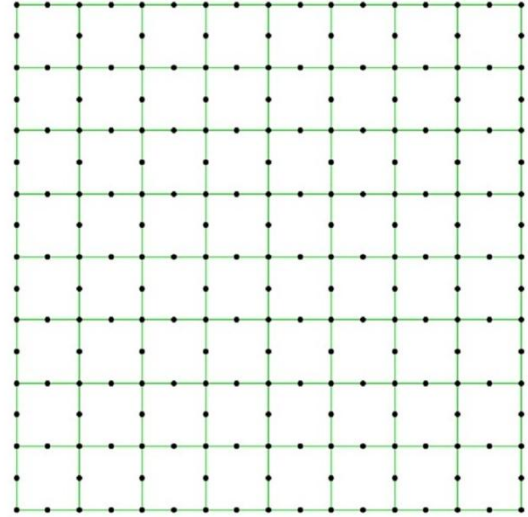
0° (FAILURE 5)



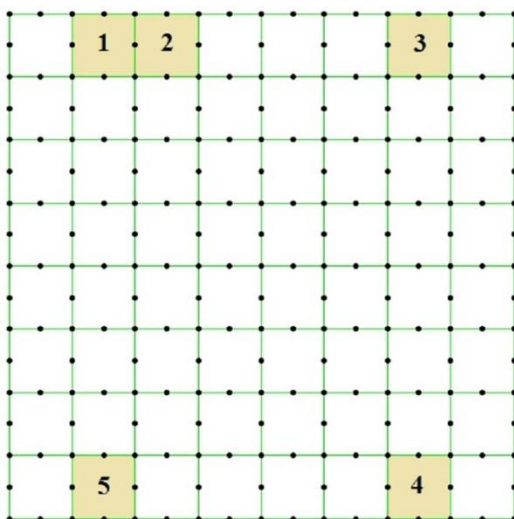
0° (0.25 UPF)



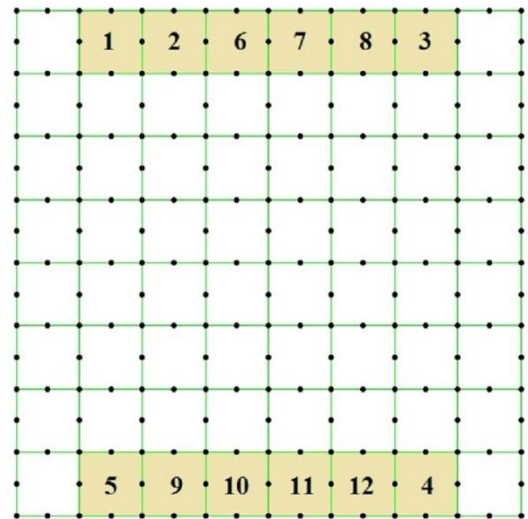
90° (FAILURE 5)



90° (0.25 UPF)

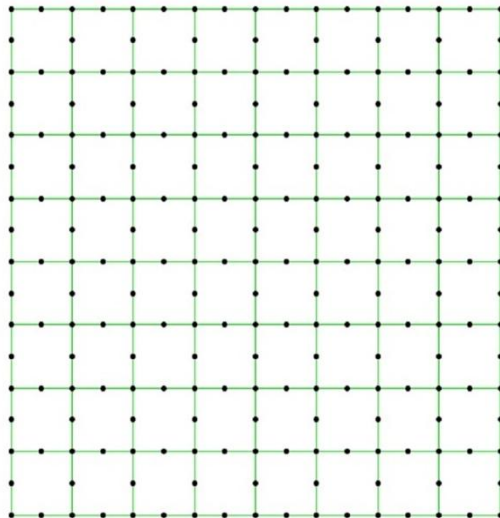


0° (FAILURE 5)

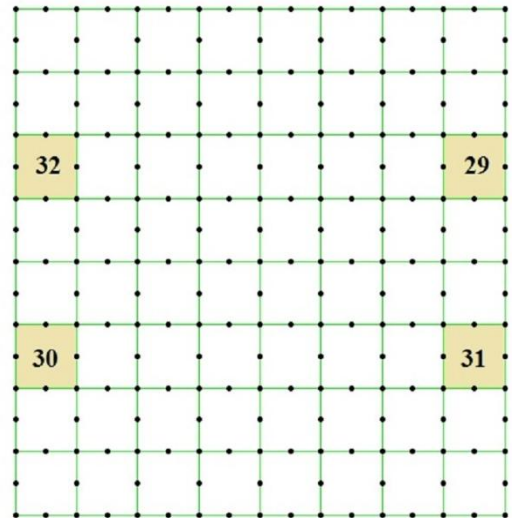


0° (0.25 UPF)

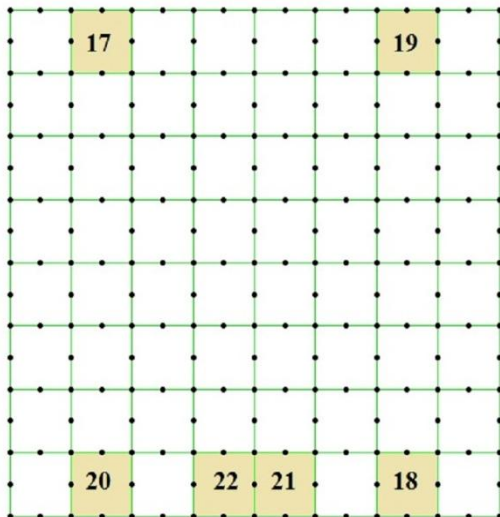
Fig. 7.4 Failure pattern of type: SYCP lamina for boundary condition: CCCC (contd.)



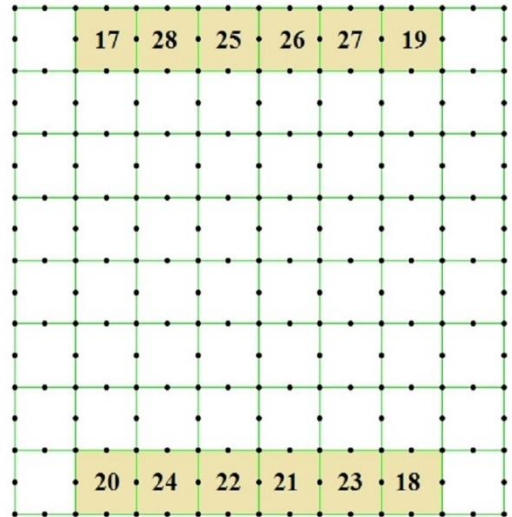
0° (0.5 UPF)



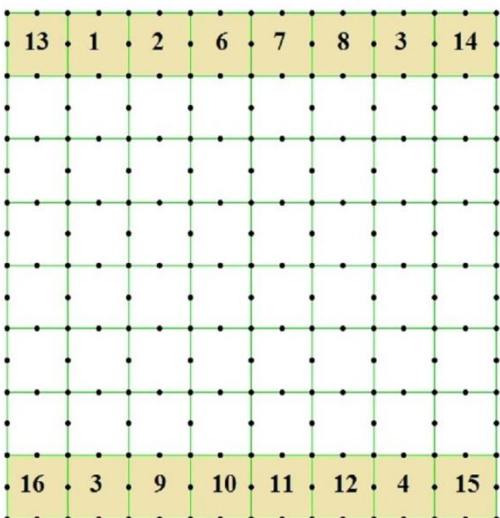
0° (0.75 UPF)



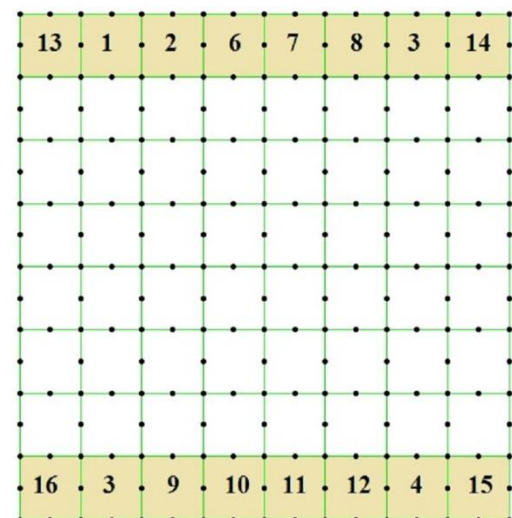
90° (0.5 UPF)



90° (0.75 UPF)

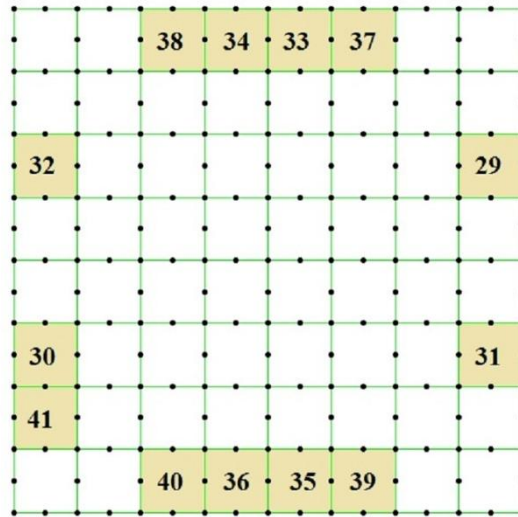


0° (0.5 UPF)

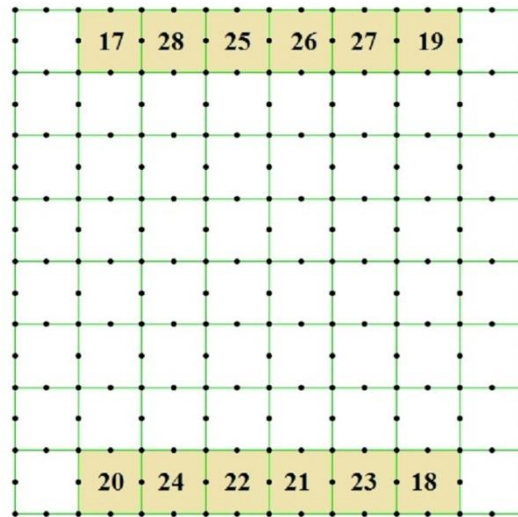


0° (0.75 UPF)

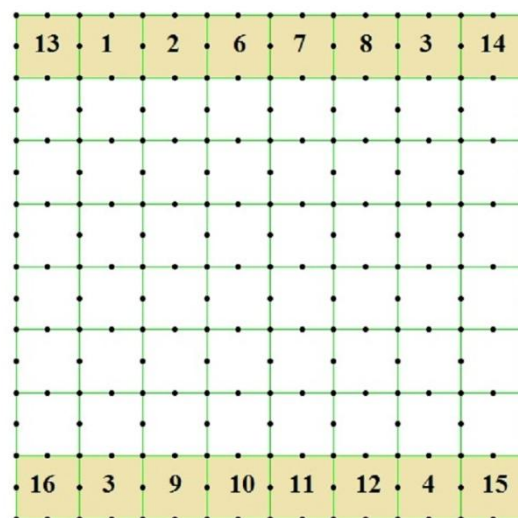
Fig. 7.4 Failure pattern of type: SYCP lamina for boundary condition: CCCC (contd.)



0° (UPF)

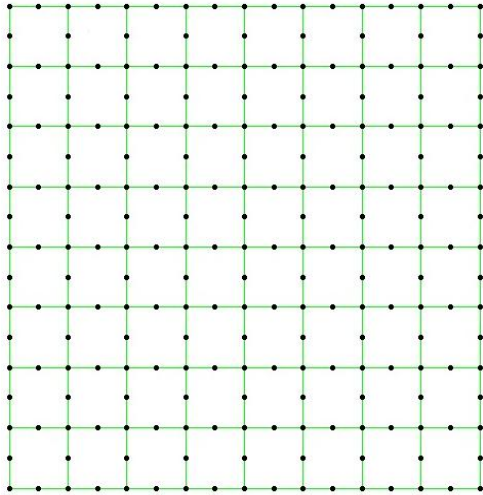


90° (UPF)

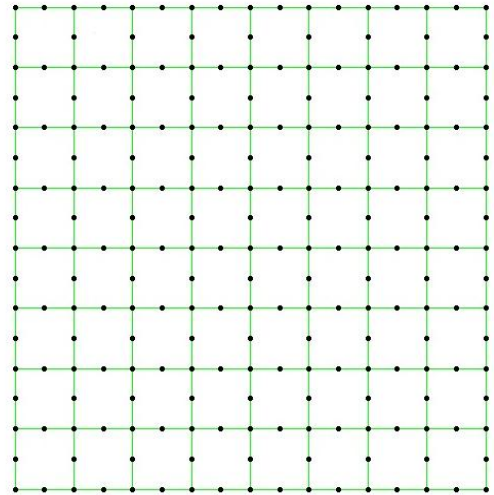


0° (UPF)

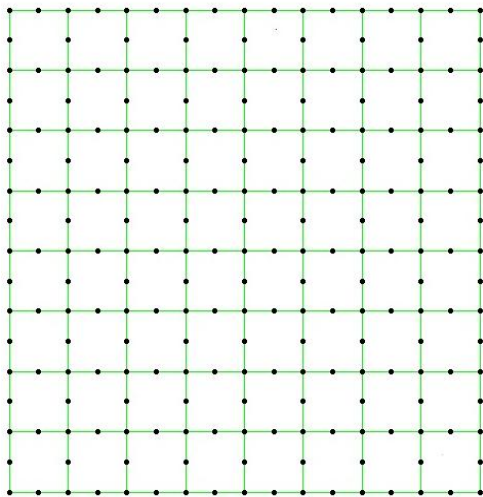
Fig. 7.5 Failure pattern of type: SYAP lamina for boundary condition: CCCC



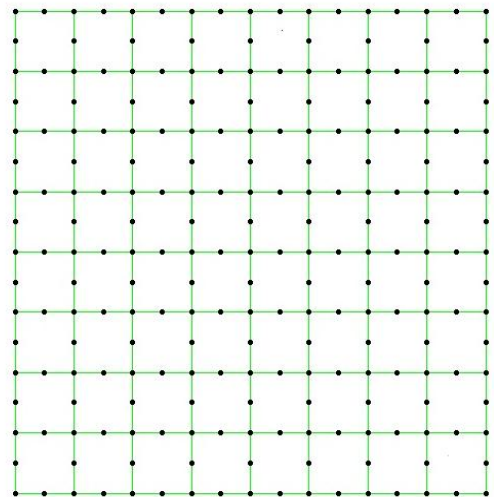
45° (FAILURE 1)



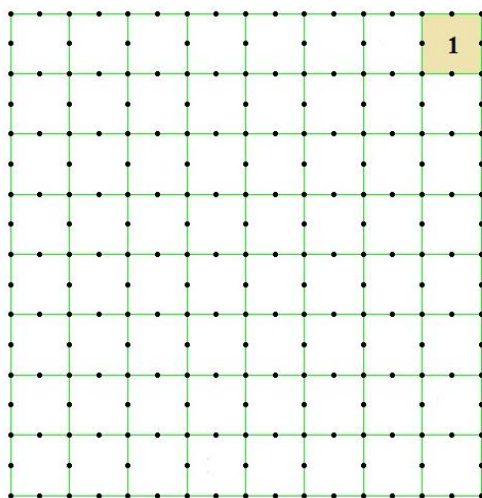
45° (FAILURE 2)



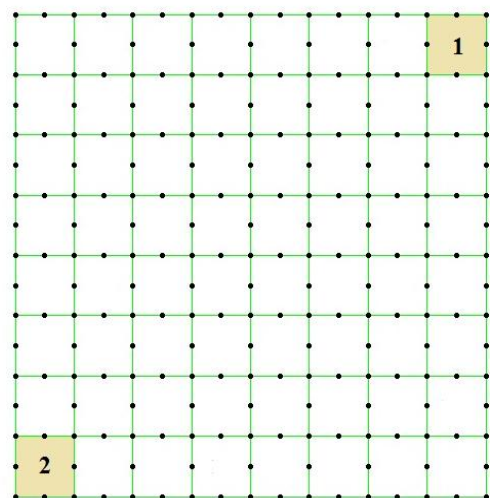
- 45° (FAILURE 1)



- 45° (FAILURE 2)

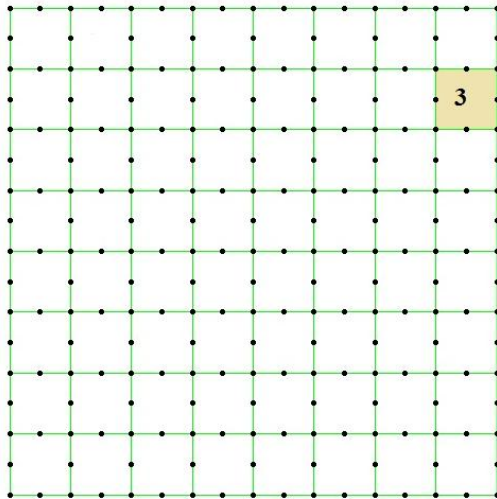


45° (FAILURE 1)

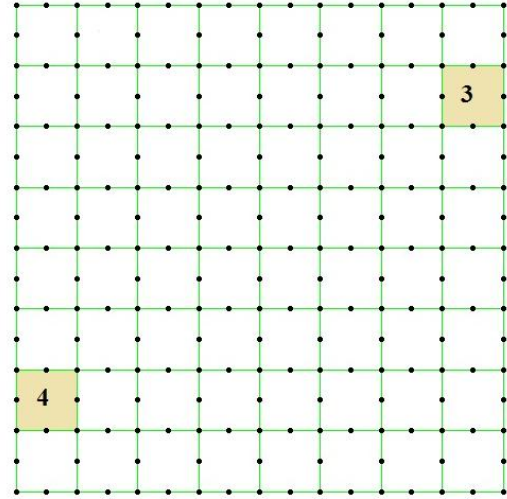


45° (FAILURE 2)

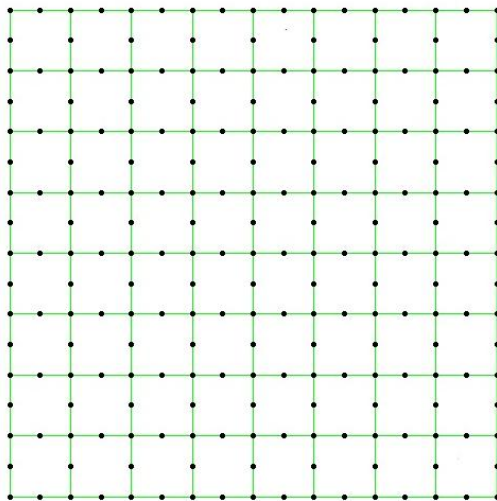
Fig. 7.5 Failure pattern of type: SYAP lamina for boundary condition: CCCC (contd.)



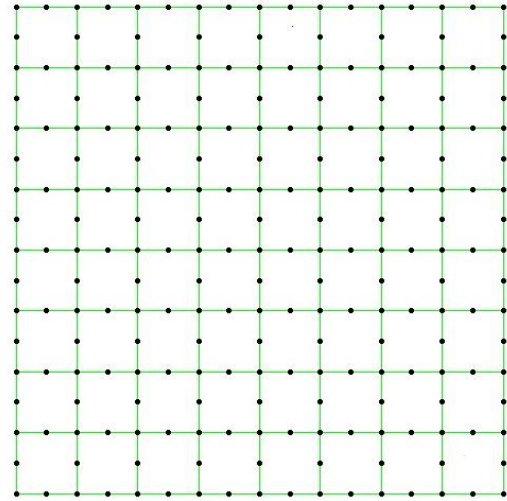
45° (FAILURE 3)



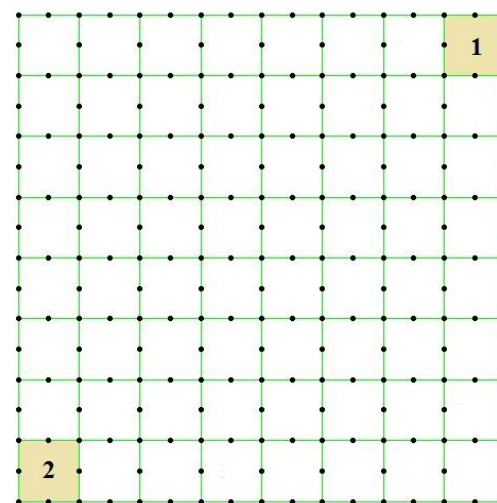
45° (FAILURE 4)



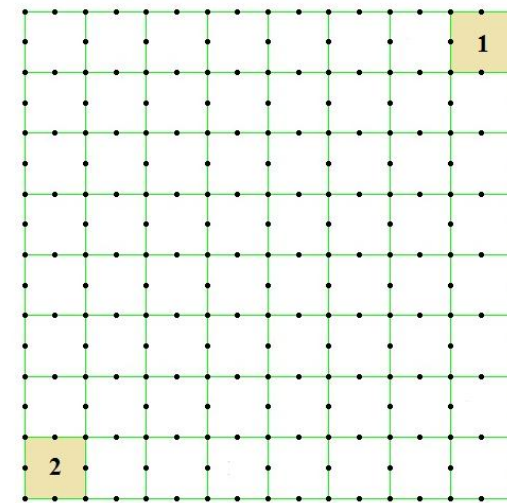
-45° (FAILURE 3)



-45° (FAILURE 4)

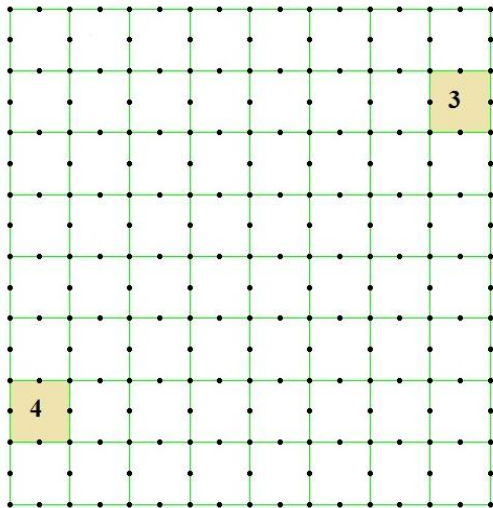


45° (FAILURE 3)

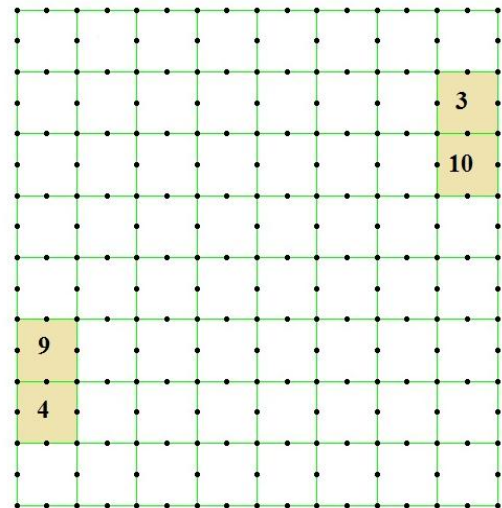


45° (FAILURE 4)

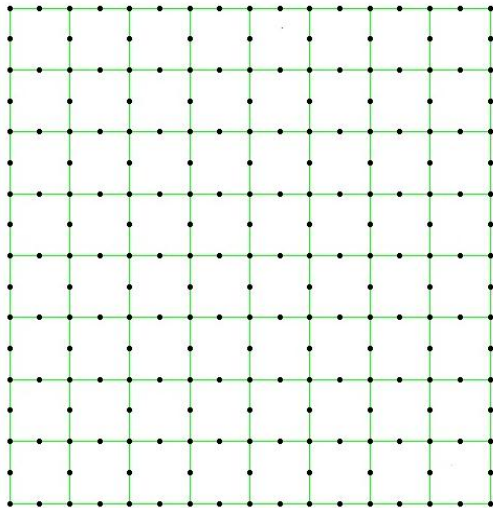
Fig. 7.5 Failure pattern of type: SYAP lamina for boundary condition: CCCC (contd.)



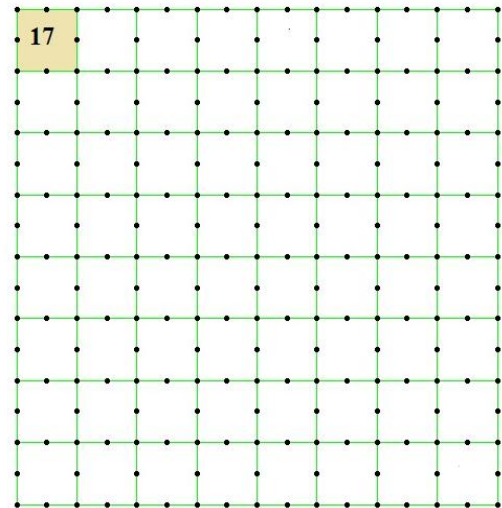
45° (FAILURE 5)



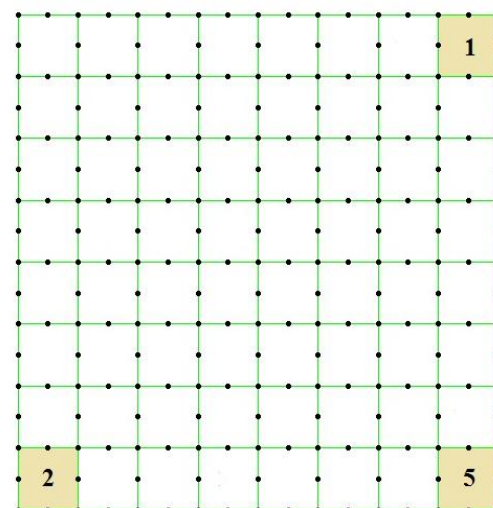
45° (0.25 UPF)



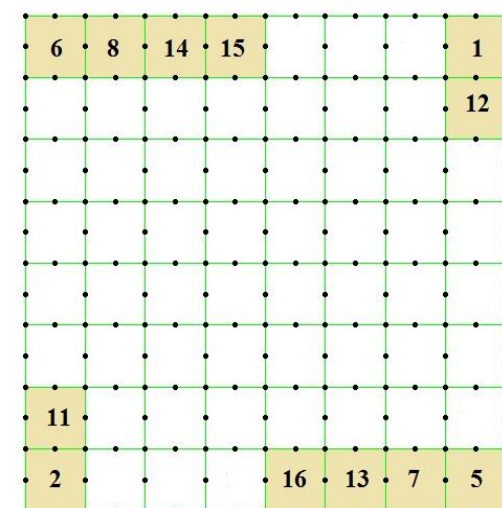
- 45° (FAILURE 5)



- 45° (0.25 UPF)

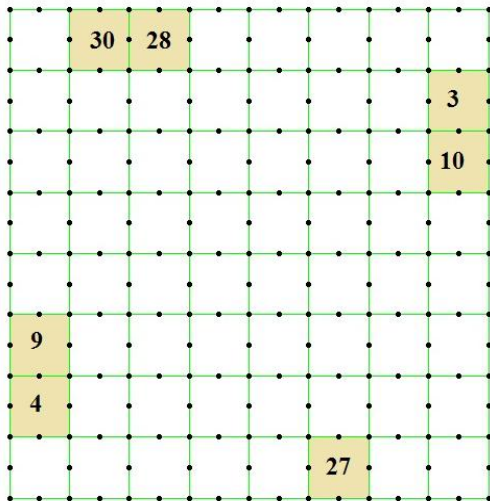


45° (FAILURE 5)

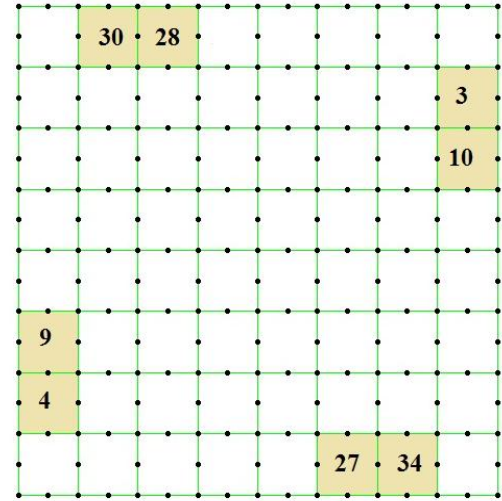


45° (0.25 UPF)

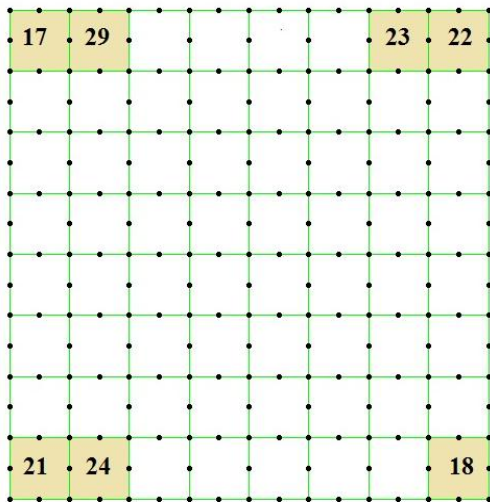
Fig. 7.5 Failure pattern of type: SYAP lamina for boundary condition: CCCC (contd.)



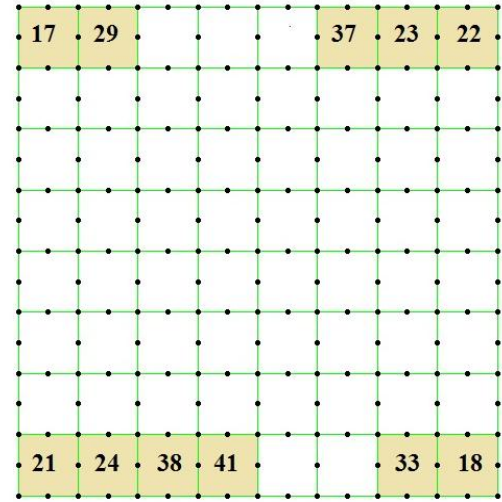
45° (0.5 UPF)



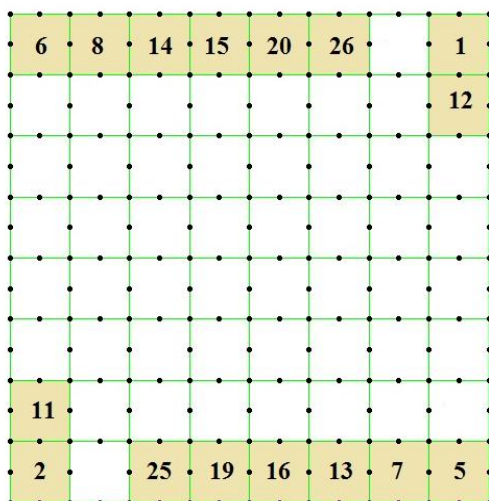
45° (0.75 UPF)



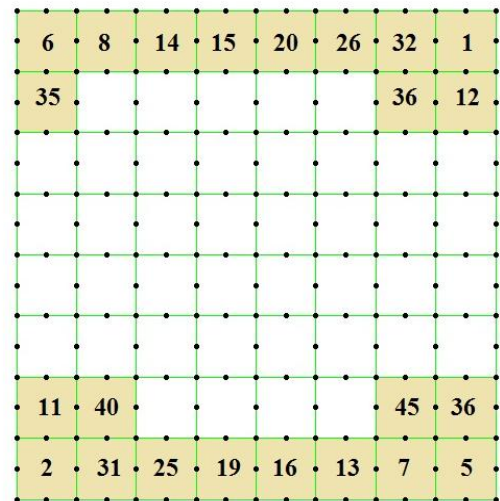
- 45° (0.5 UPF)



- 45° (0.75 UPF)

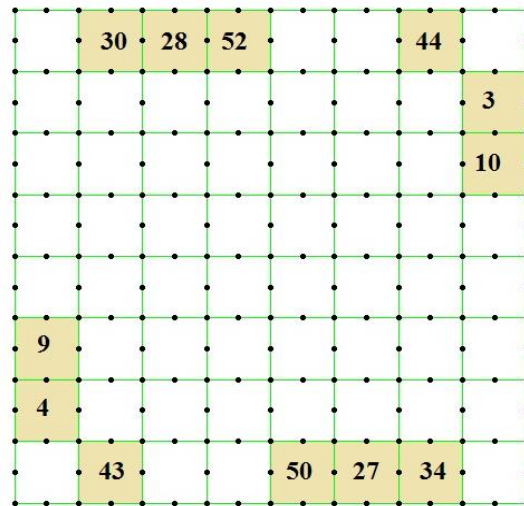


45° (0.5 UPF)

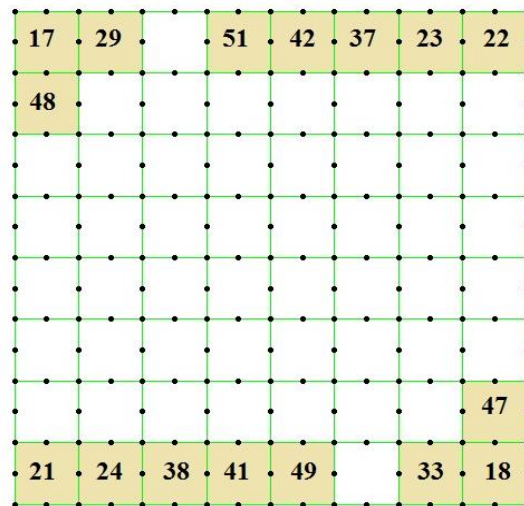


45° (0.75 UPF)

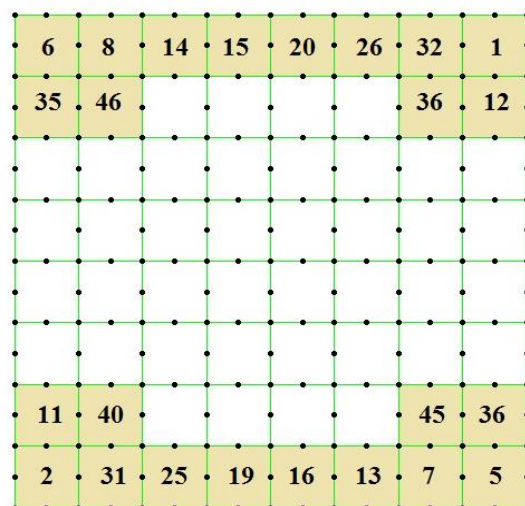
Fig. 7.5 Failure pattern of type: SYAP lamina for boundary condition: CCCC (contd.)



45° (UPF)

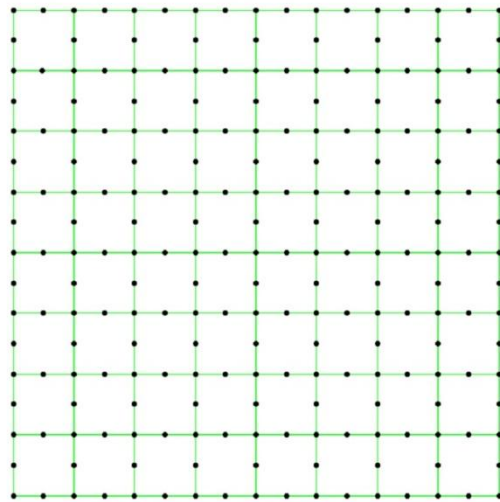


- 45° (UPF)

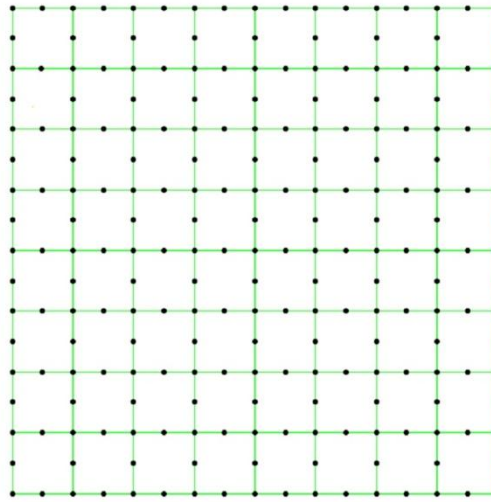


45° (UPF)

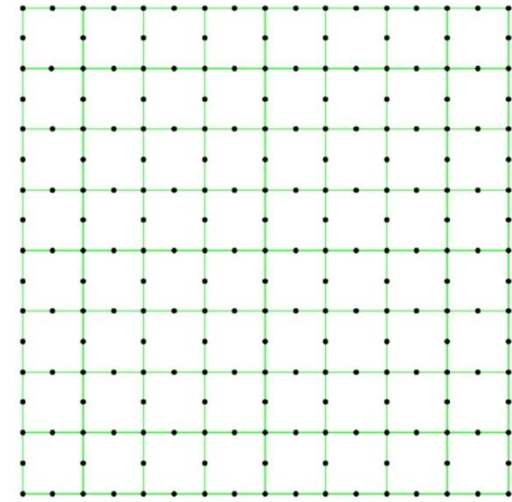
Fig. 7.6 Failure pattern of type: ASCP lamina for boundary condition: CSSC



0° (FAILURE 1)



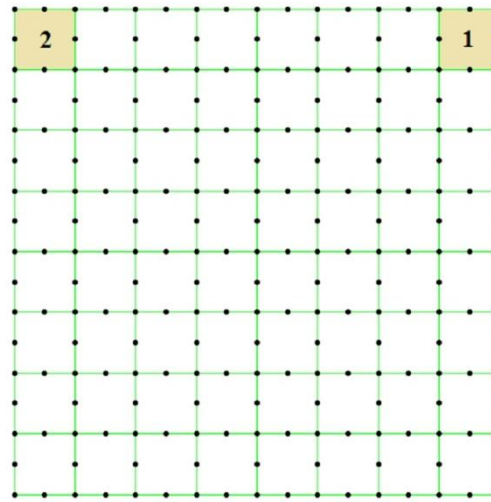
0° (FAILURE 2)



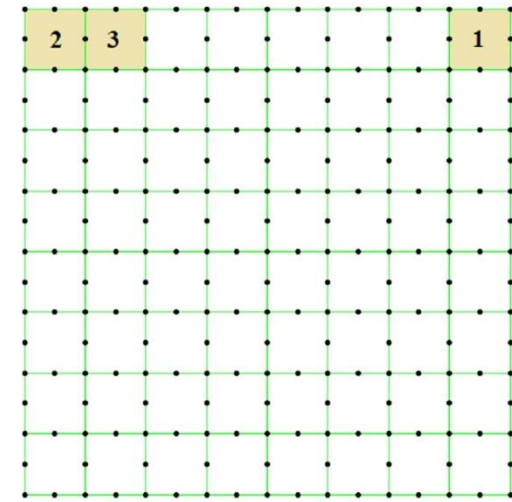
0° (FAILURE 3)



90° (FAILURE 1)

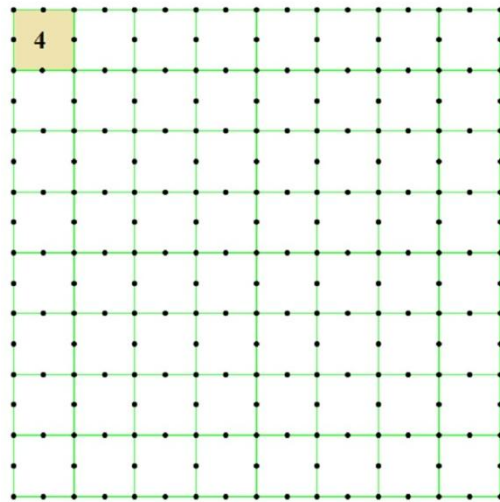


90° (FAILURE 2)

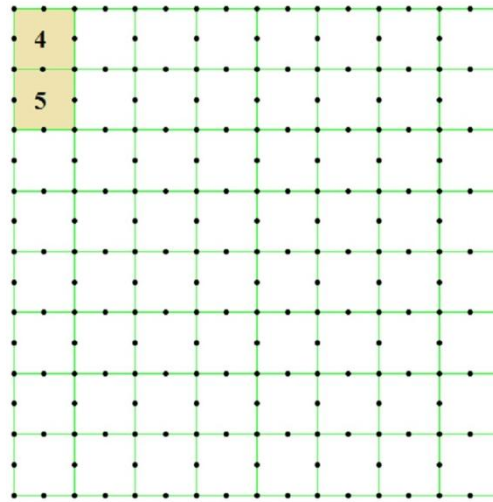


90° (FAILURE 3)

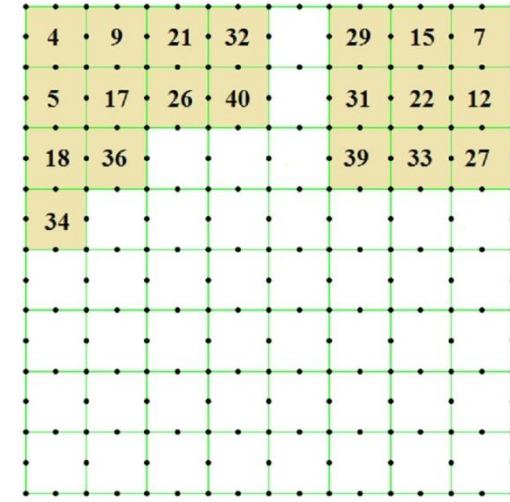
Fig. 7.6 Failure pattern of type: ASCP lamina for boundary condition: CSSC (contd.)



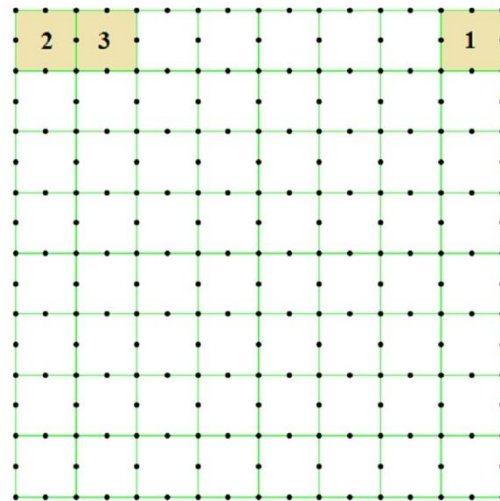
0° (FAILURE 4)



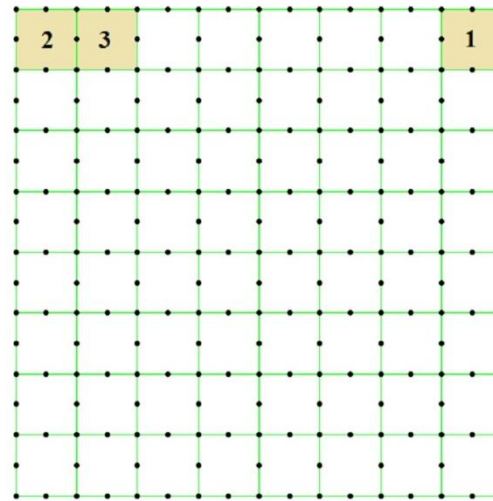
0° (FAILURE 5)



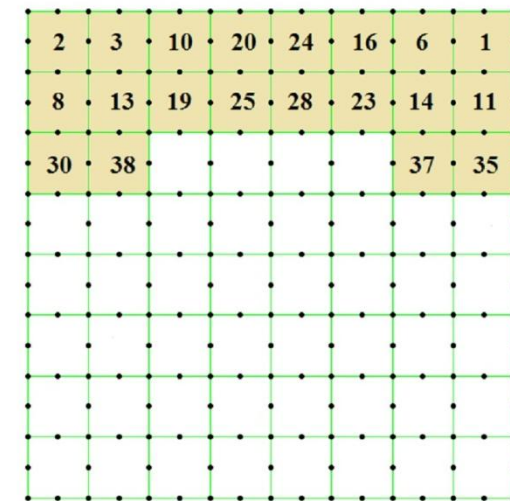
0° (0.25 UPF)



90° (FAILURE 4)

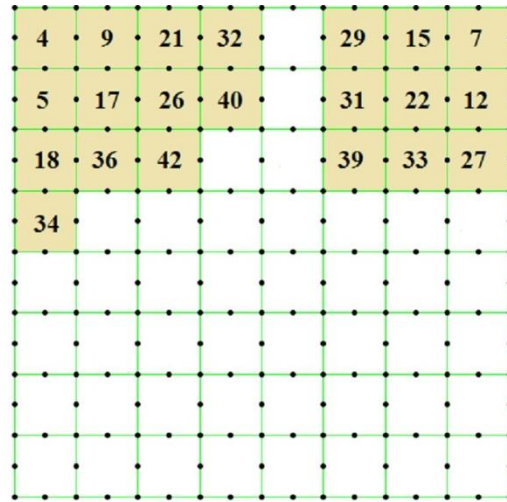


90° (FAILURE 5)

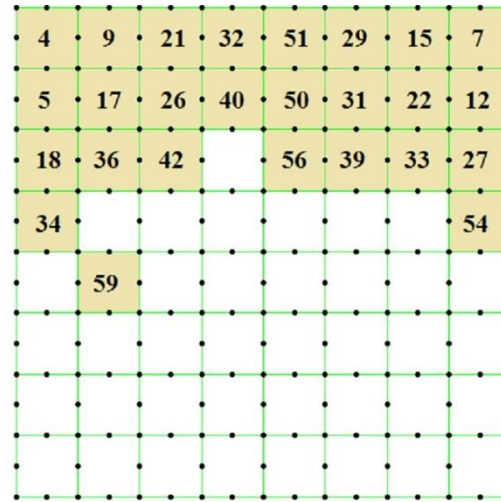


90° (0.25 UPF)

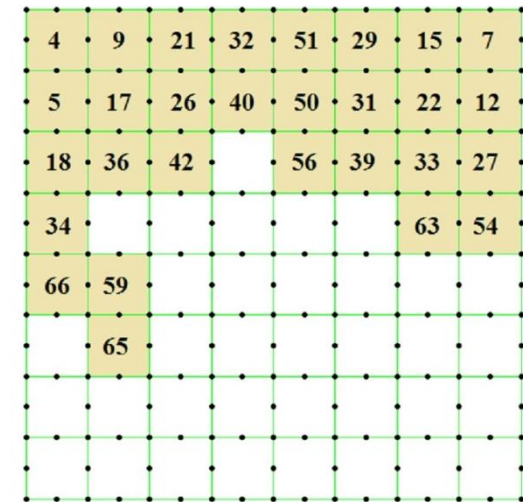
Fig. 7.6 Failure pattern of type: ASCP lamina for boundary condition: CSSC (contd.)



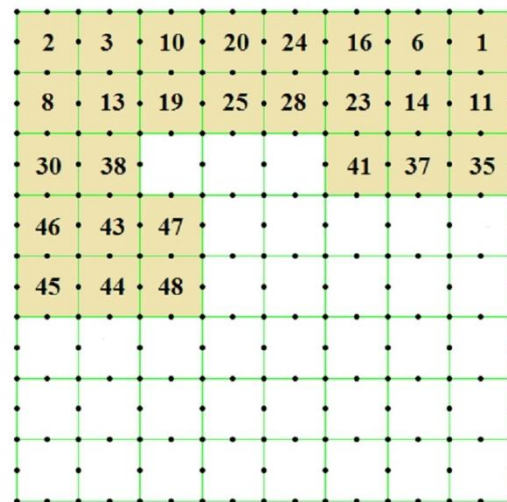
0° (0.5 UPF)



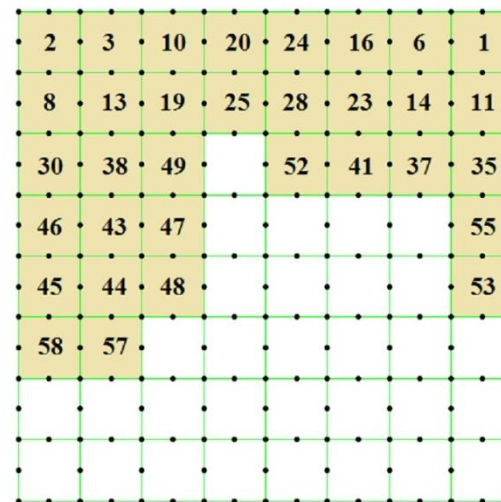
0° (0.75 UPF)



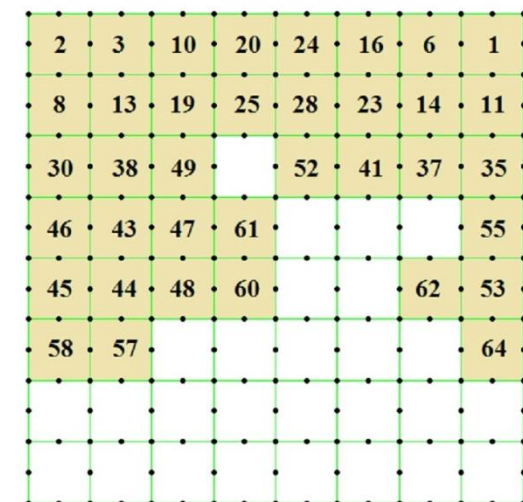
0° (UPF)



90° (0.5 UPF)

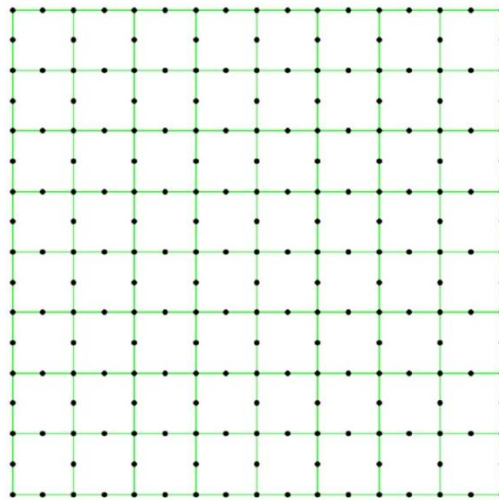


90° (0.75 UPF)

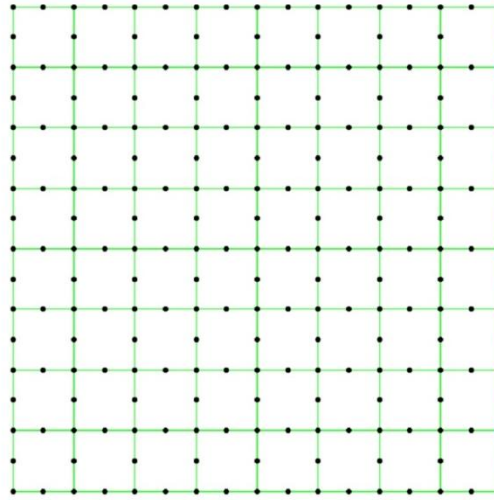


90° (UPF)

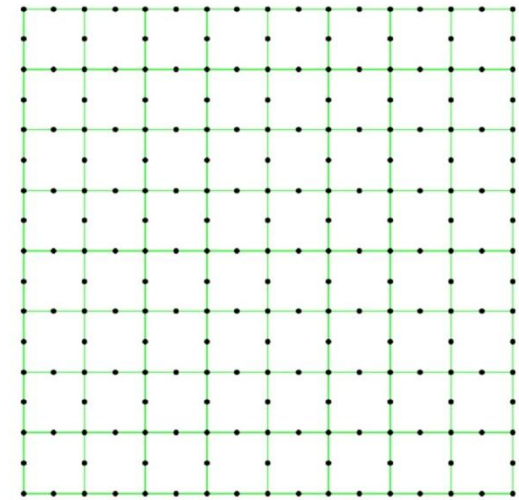
Fig. 7.7 Failure pattern of type: ASAP lamina for boundary condition: CSSC



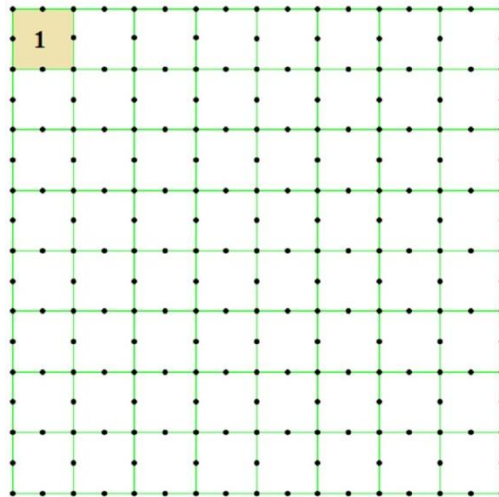
45° (FAILURE 1)



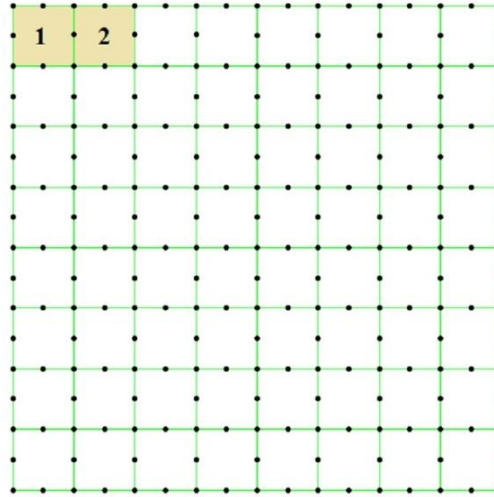
45° (FAILURE 2)



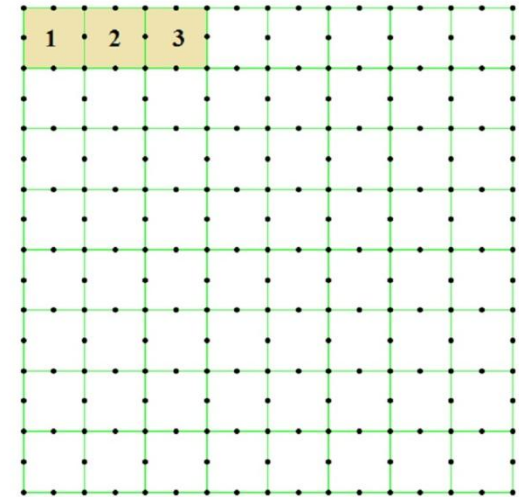
45° (FAILURE 3)



- 45° (FAILURE 1)

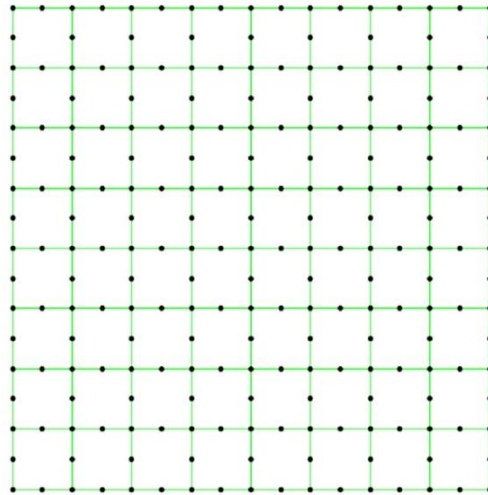


- 45° (FAILURE 2)

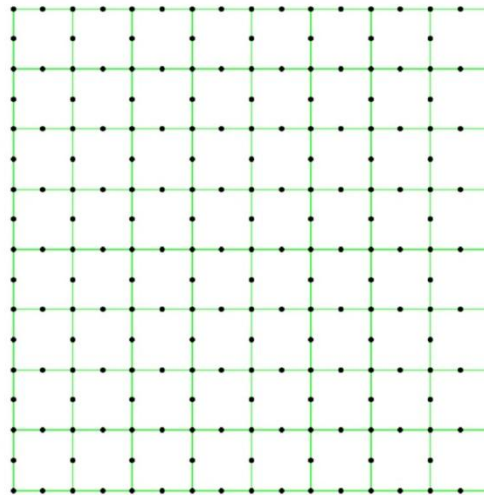


- 45° (FAILURE 3)

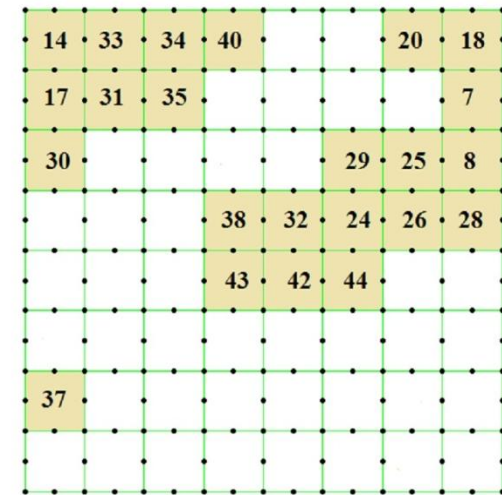
Fig. 7.7 Failure pattern of type: ASAP lamina for boundary condition: CSSC (Contd.)



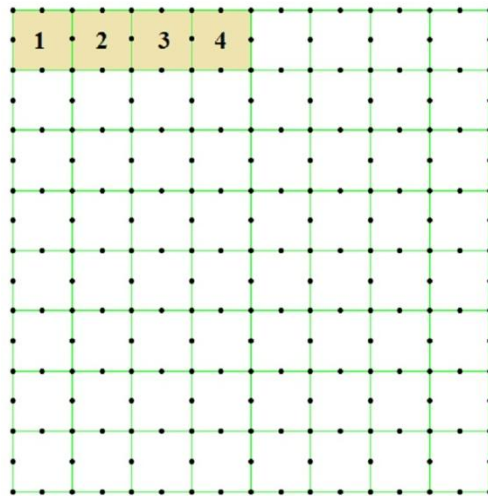
45° (FAILURE 4)



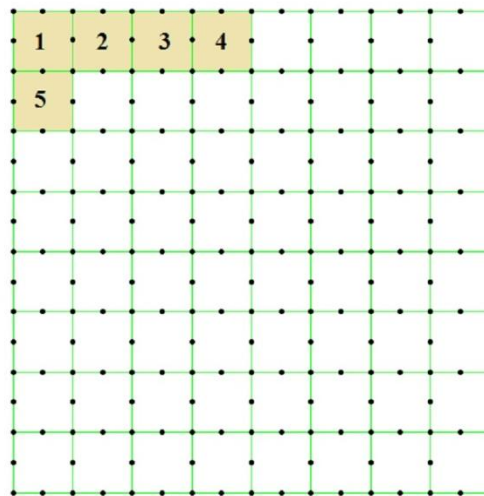
45° (FAILURE 5)



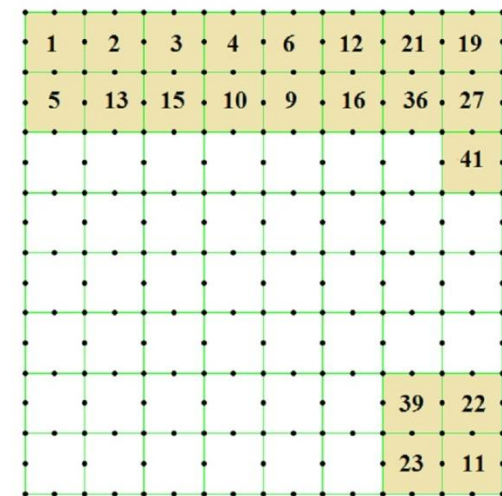
45° (0.25 UPF)



- 45° (FAILURE 4)

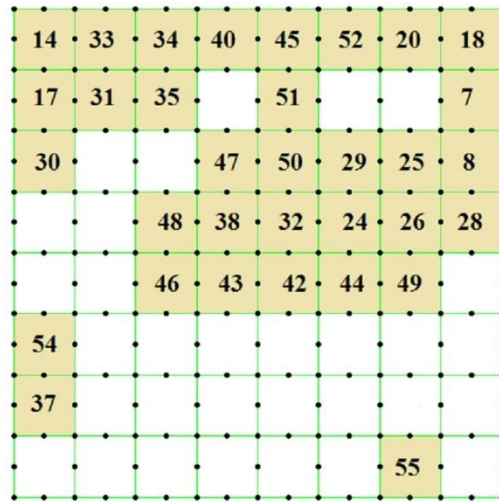


- 45° (FAILURE 5)

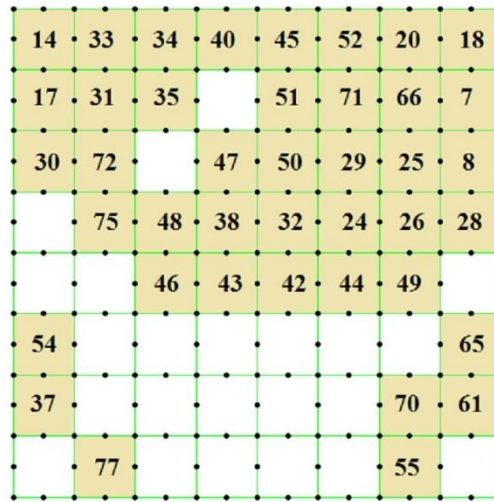


- 45° (0.25 UPF)

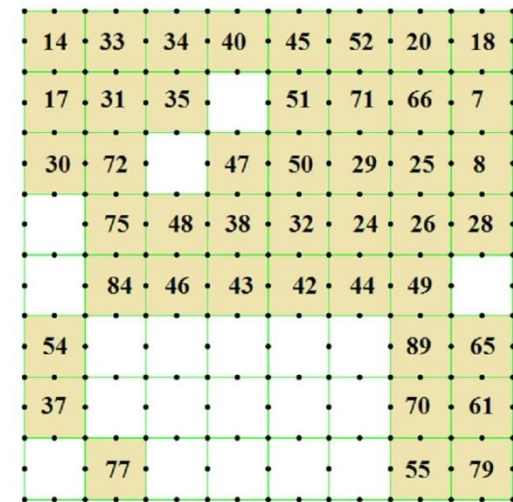
Fig. 7.7 Failure pattern of type: ASAP lamina for boundary condition: CSSC (Contd.)



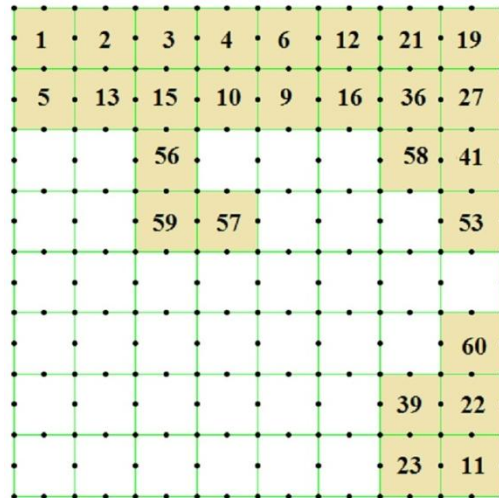
45° (0.5 UPF)



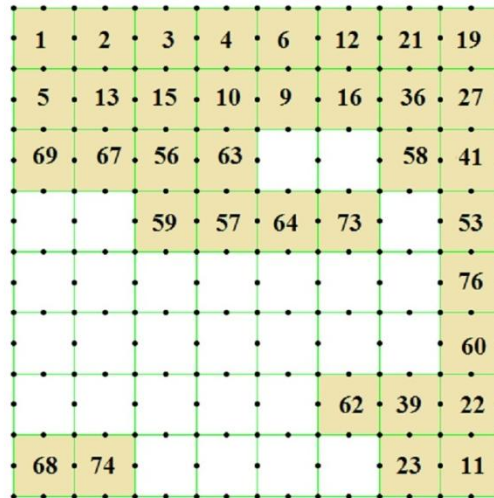
45° (0.75 UPF)



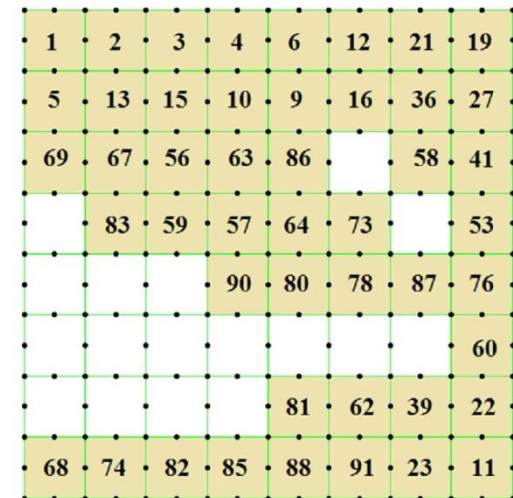
45° (UPF)



- 45° (0.5 UPF)

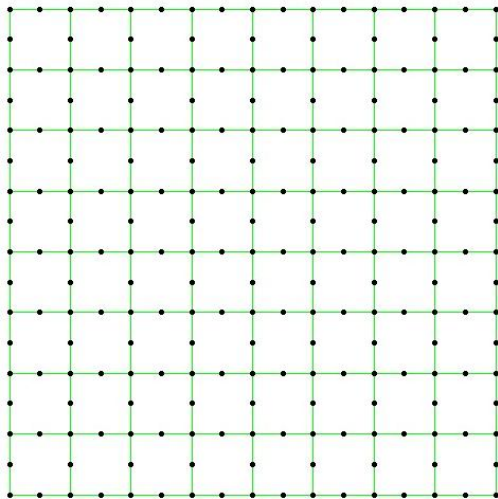


- 45° (0.75 UPF)

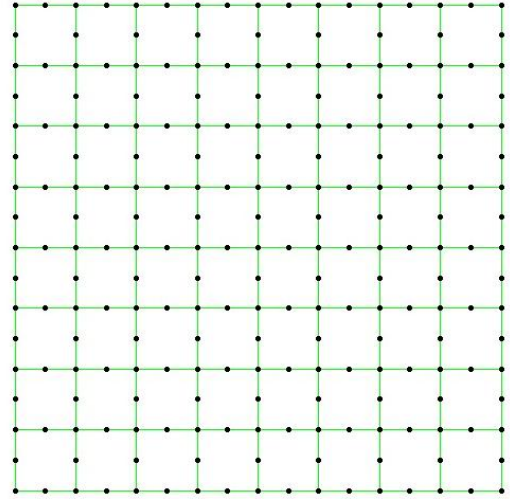


- 45° (UPF)

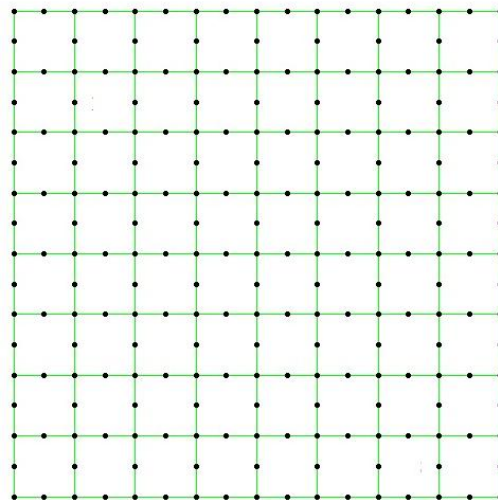
Fig. 7.8 Failure pattern of type: SYCP lamina for boundary condition: CSSC



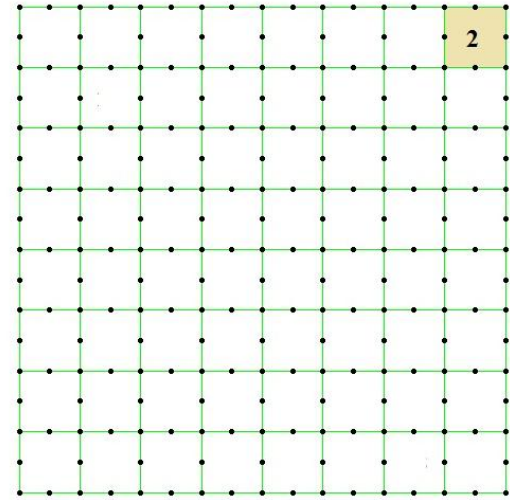
0° (FAILURE 1)



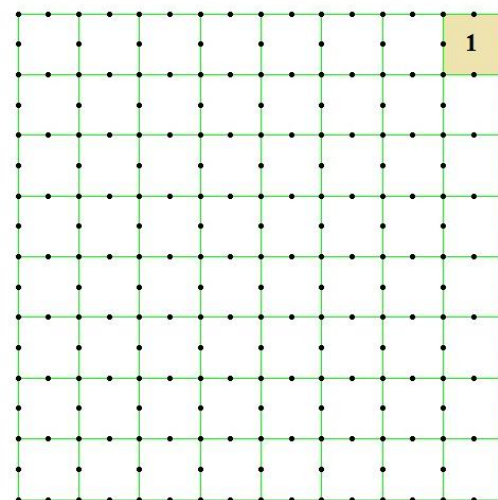
0° (FAILURE 2)



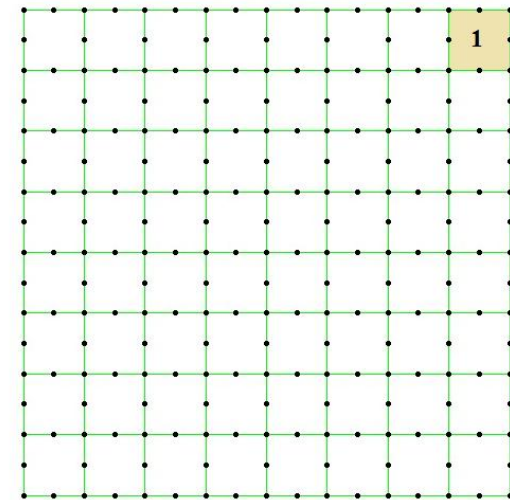
90° (FAILURE 1)



90° (FAILURE 2)

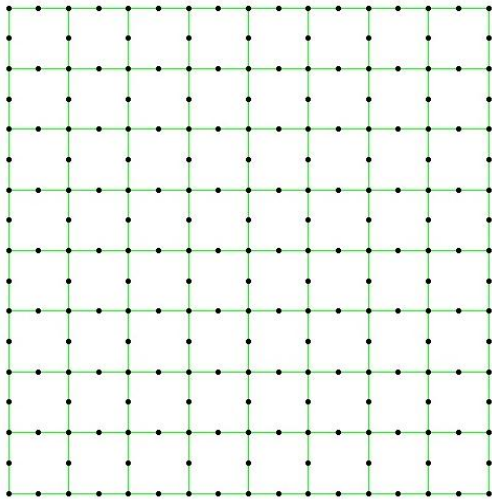


0° (FAILURE 1)

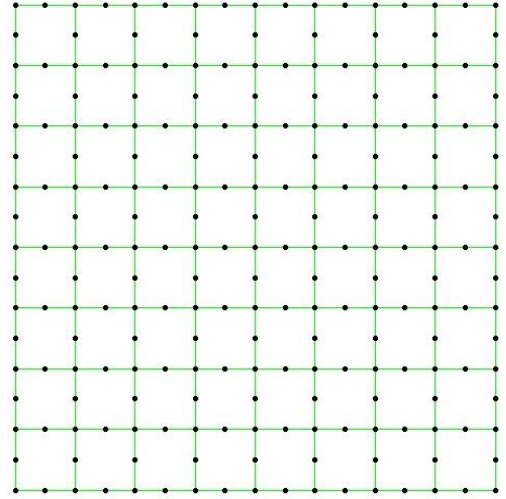


0° (FAILURE 2)

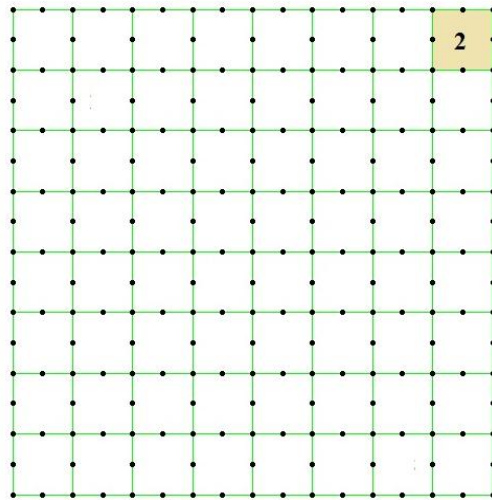
Fig. 7.8 Failure pattern of type: SYCP lamina for boundary condition: CSSC (Contd.)



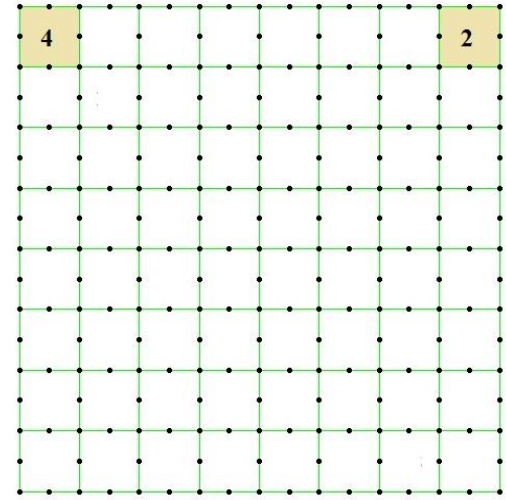
0° (FAILURE 3)



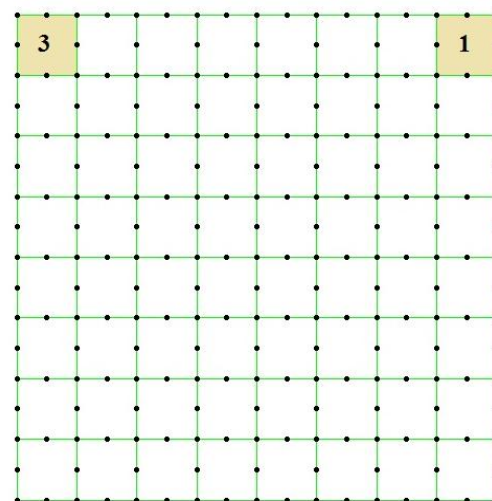
0° (FAILURE 4)



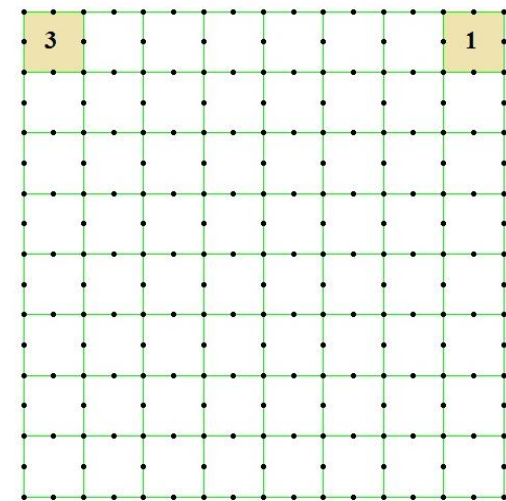
90° (FAILURE 3)



90° (FAILURE 4)

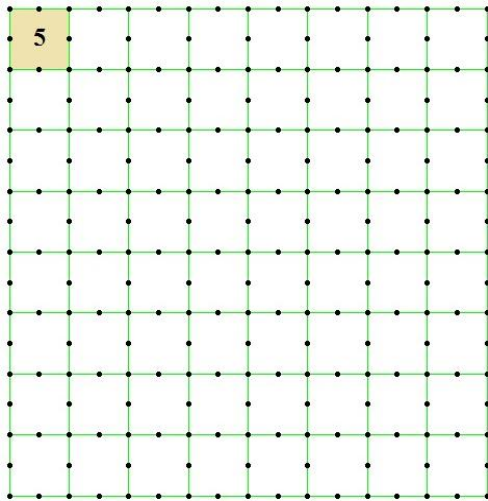


0° (FAILURE 3)

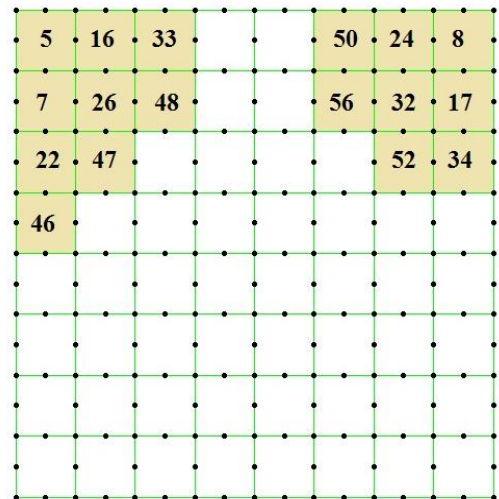


0° (FAILURE 4)

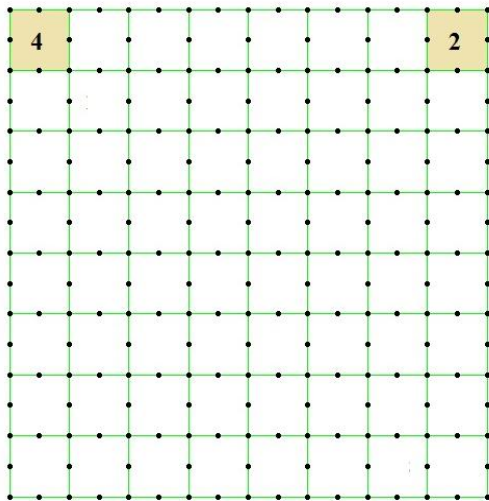
Fig. 7.8 Failure pattern of type: SYCP lamina for boundary condition: CSSC (Contd.)



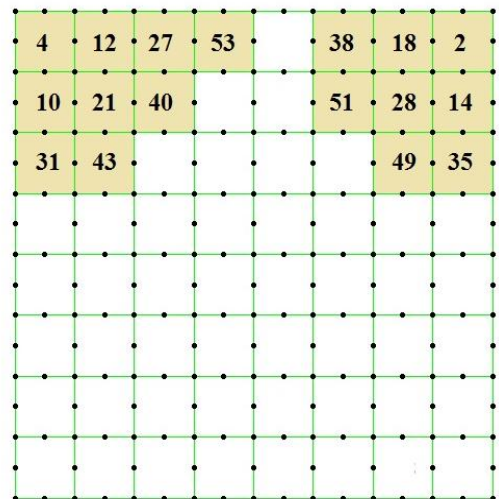
0° (FAILURE 5)



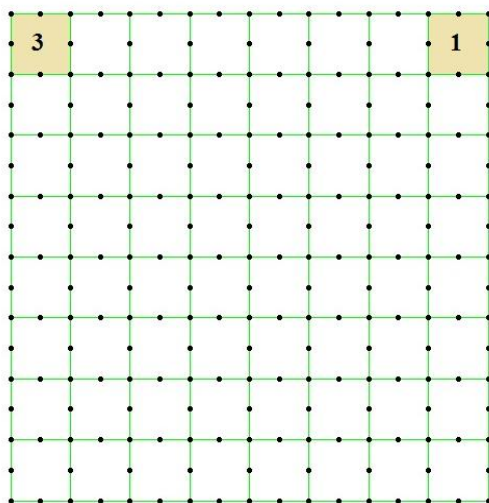
0° (0.25 UPF)



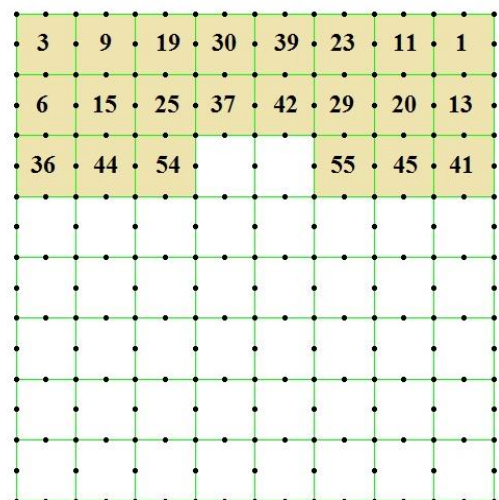
90° (FAILURE 5)



90° (0.25 UPF)

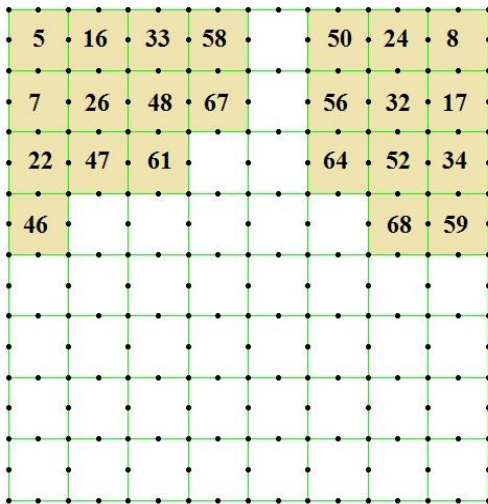


0° (FAILURE 5)

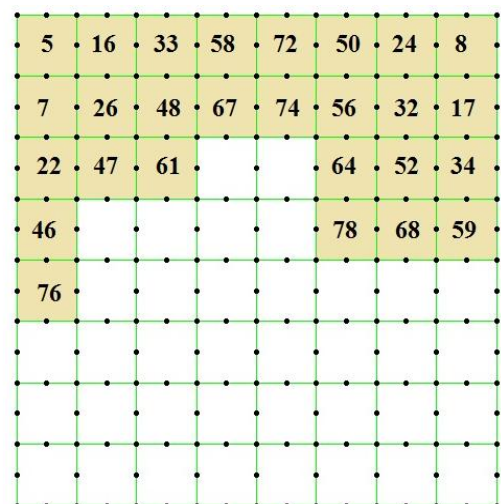


0° (0.25 UPF)

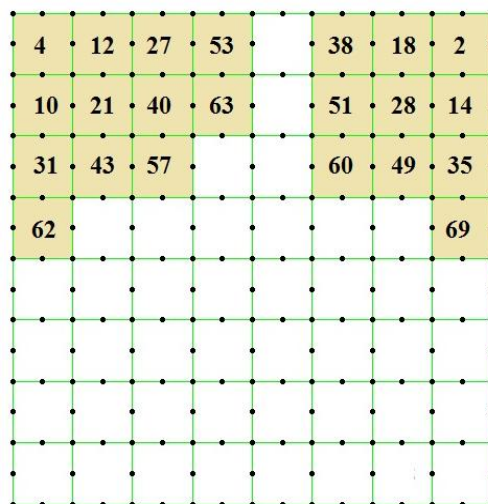
Fig. 7.8 Failure pattern of type: SYCP lamina for boundary condition: CSSC (Contd.)



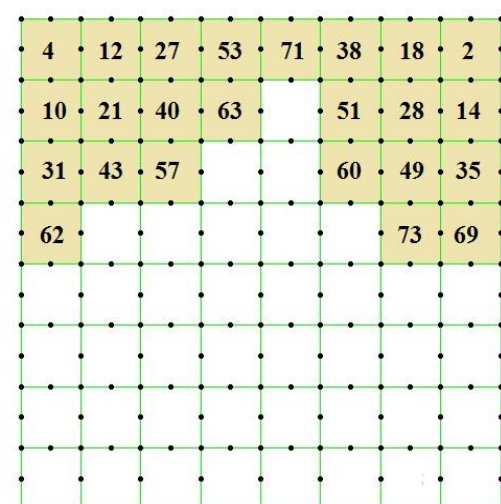
0° (0.5 UPF)



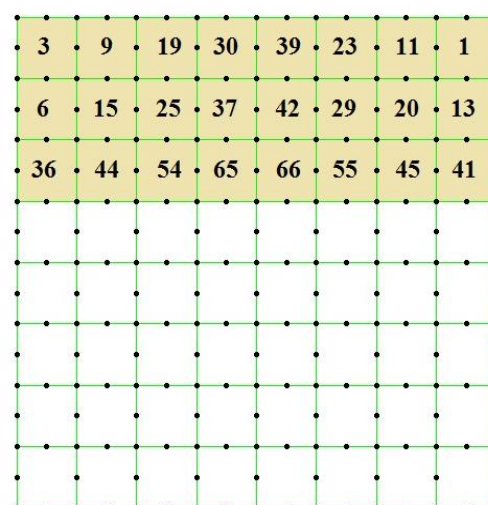
0° (0.75 UPF)



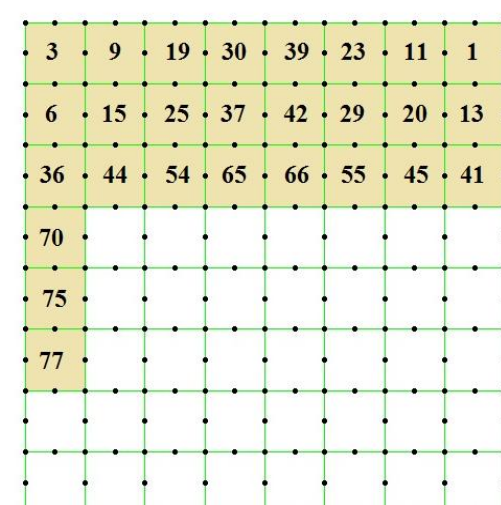
90° (0.5 UPF)



90° (0.75 UPF)



0° (0.5 UPF)



0° (0.75 UPF)

Fig. 7.8 Failure pattern of type: SYCP lamina for boundary condition: CSSC (Contd.)

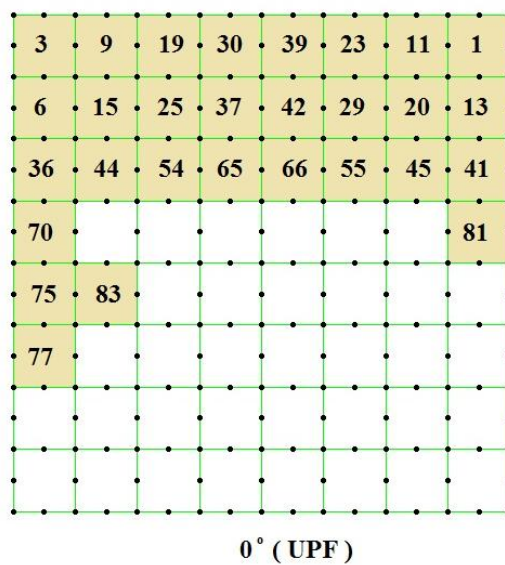
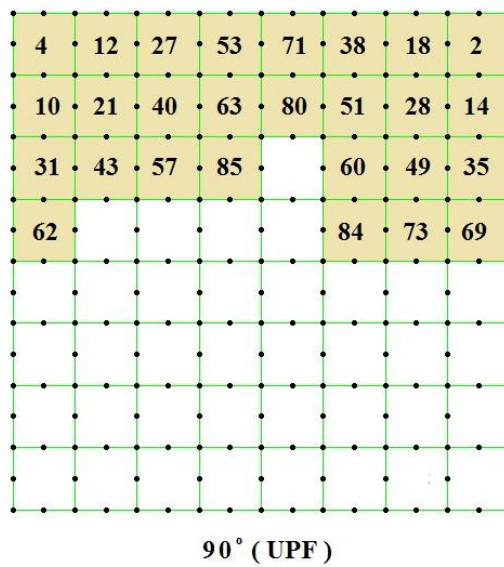
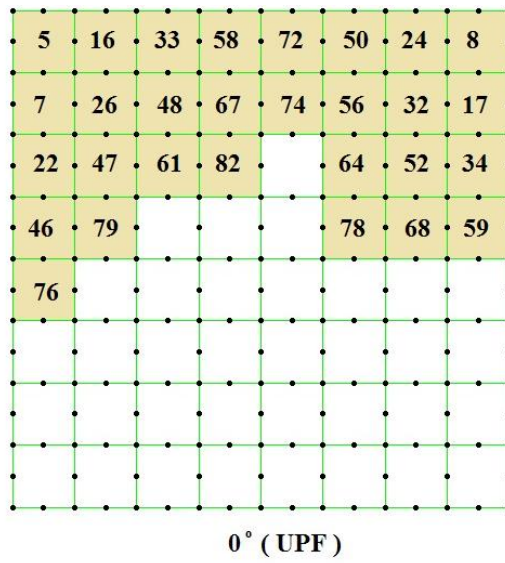
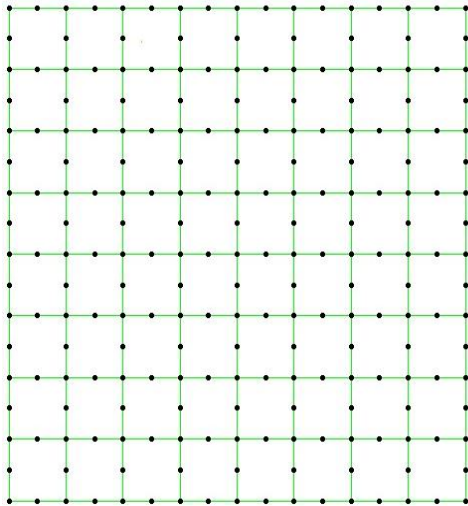
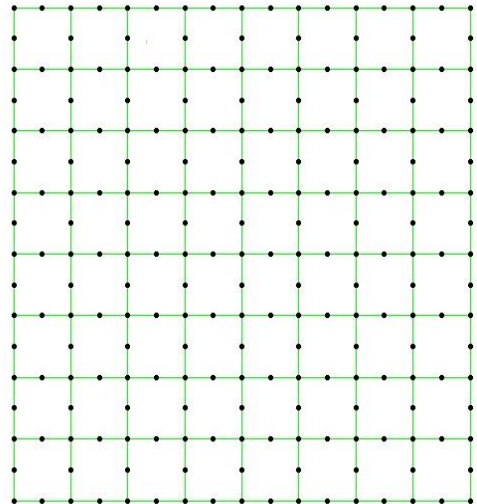


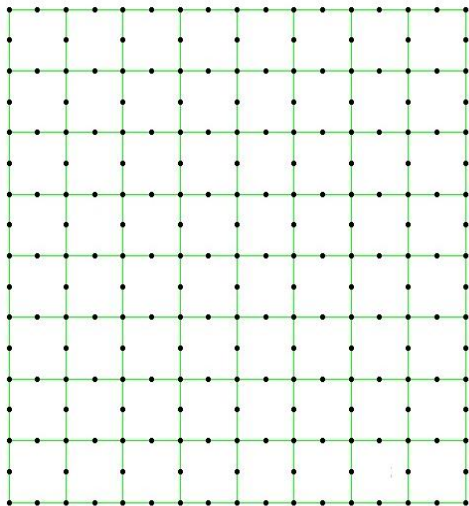
Fig. 7.9 Failure pattern of type: SYAP lamina for boundary condition: CSSC



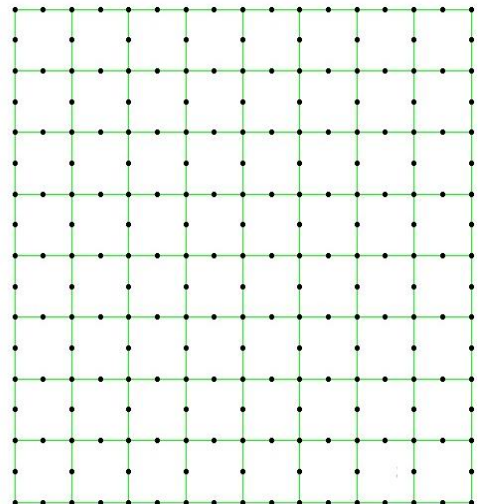
45° (FAILURE 1)



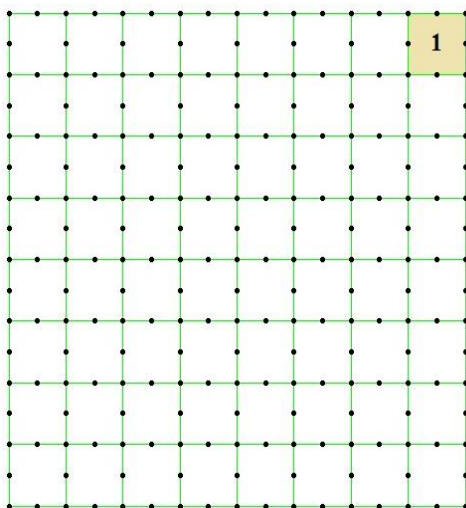
45° (FAILURE 2)



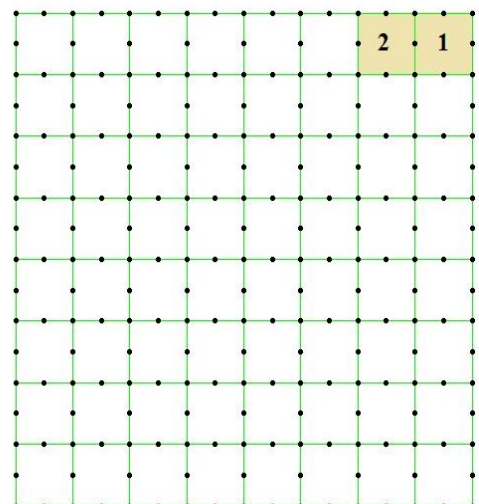
- 45° (FAILURE 1)



- 45° (FAILURE 2)

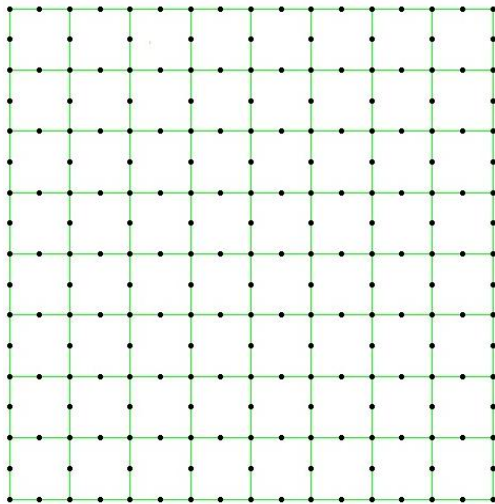


45° (FAILURE 1)

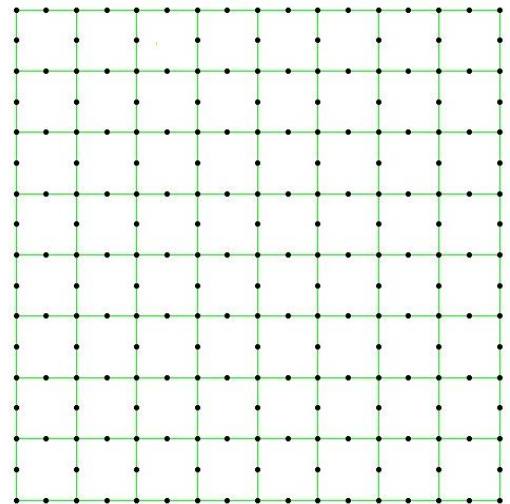


45° (FAILURE 2)

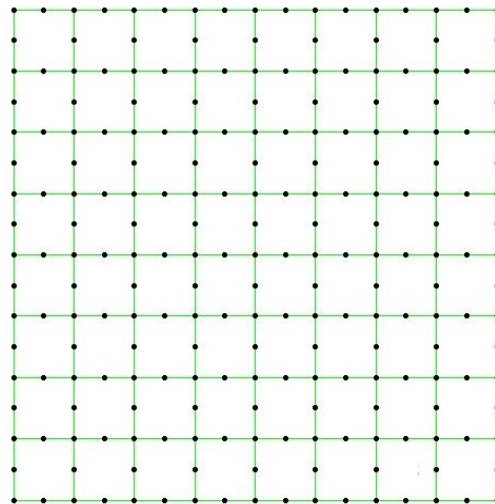
Fig. 7.9 Failure pattern of type: SYAP lamina for boundary condition: CSSC (Contd.)



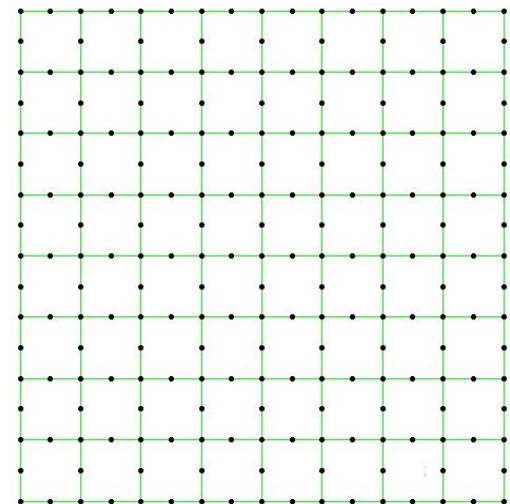
45° (FAILURE 3)



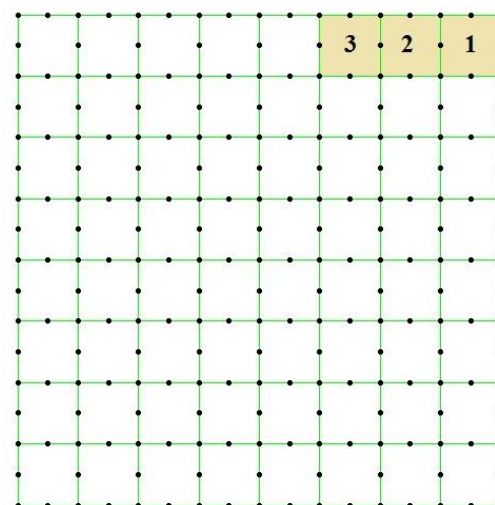
45° (FAILURE 4)



- 45° (FAILURE 3)



- 45° (FAILURE 4)

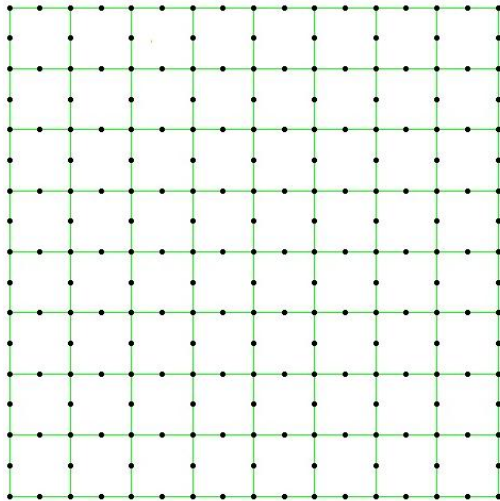


45° (FAILURE 3)

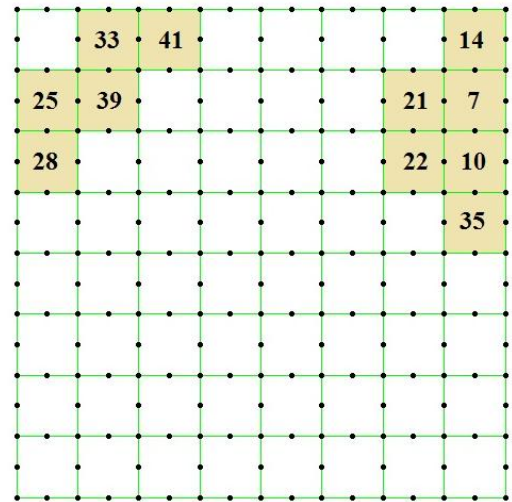


45° (FAILURE 4)

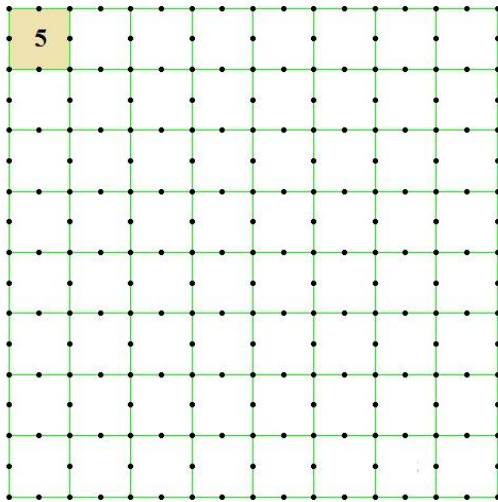
Fig. 7.9 Failure pattern of type: SYAP lamina for boundary condition: CSSC (Contd.)



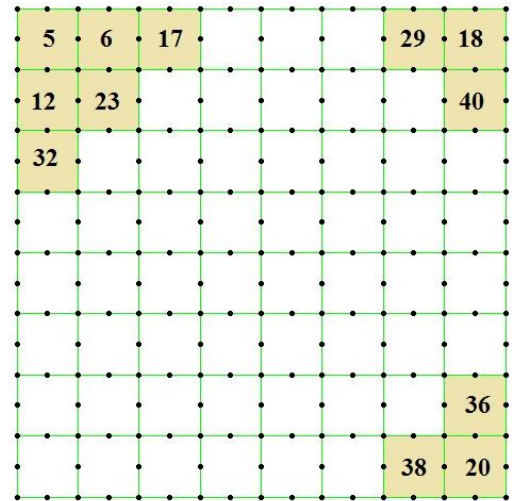
45° (FAILURE 5)



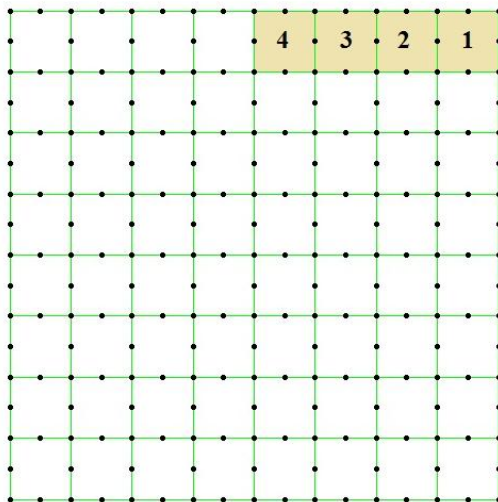
45° (0.25 UPF)



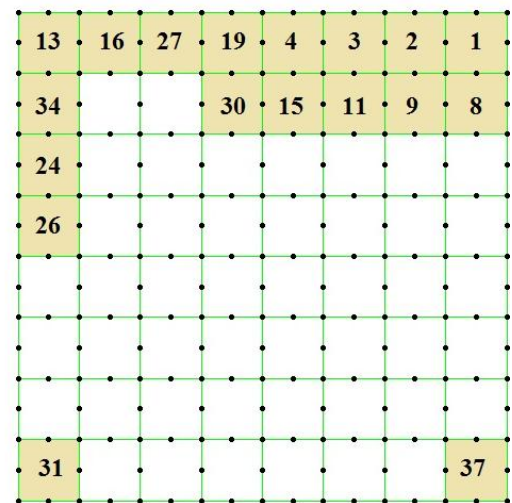
- 45° (FAILURE 5)



- 45° (0.25 UPF)

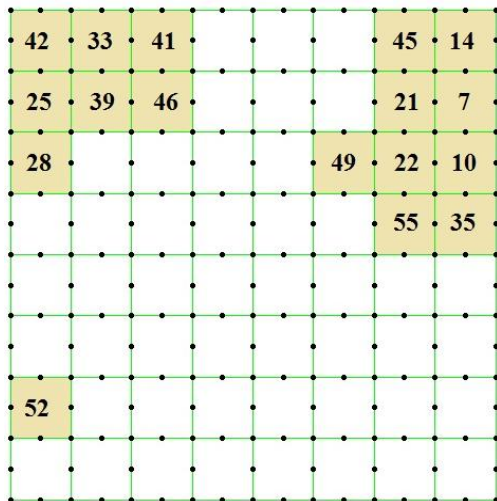


45° (FAILURE 5)

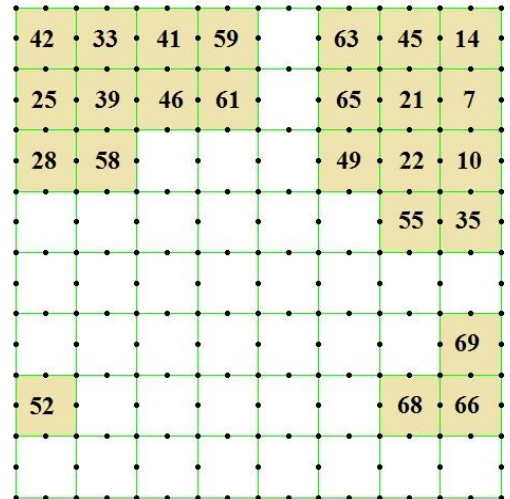


45° (0.25 UPF)

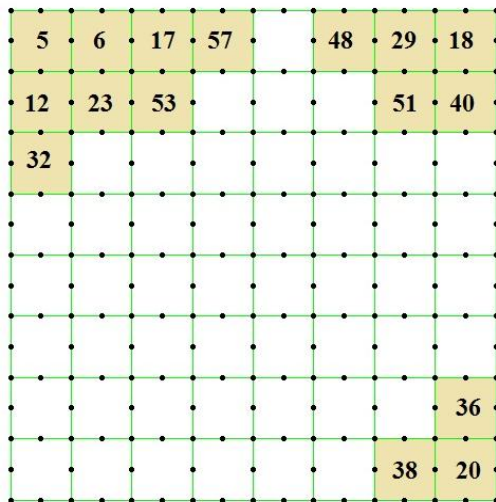
Fig. 7.9 Failure pattern of type: SYAP lamina for boundary condition: CSSC (Contd.)



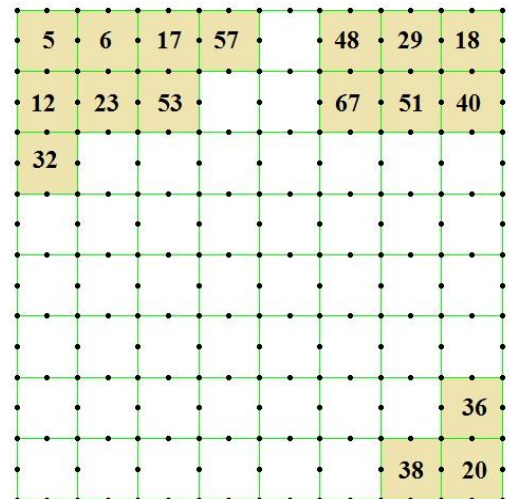
45° (0.5 UPF)



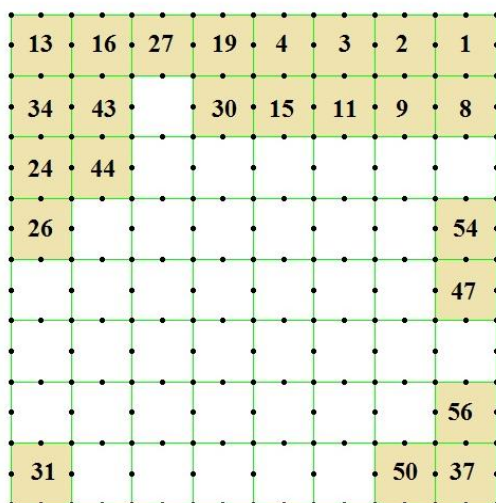
45° (0.75 UPF)



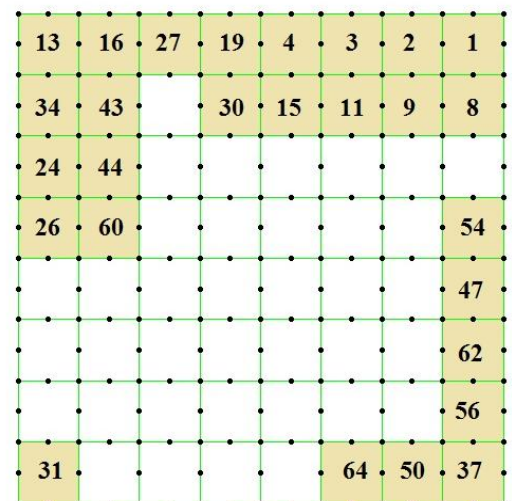
- 45° (0.5 UPF)



- 45° (0.75 UPF)

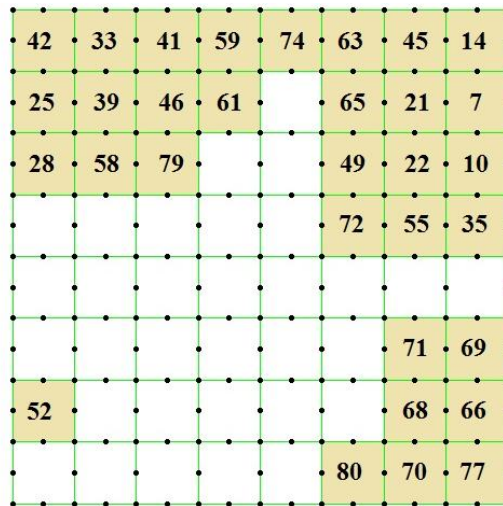


45° (0.5 UPF)

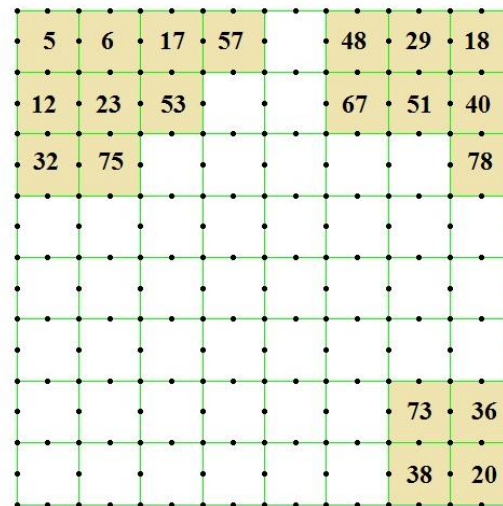


45° (0.75 UPF)

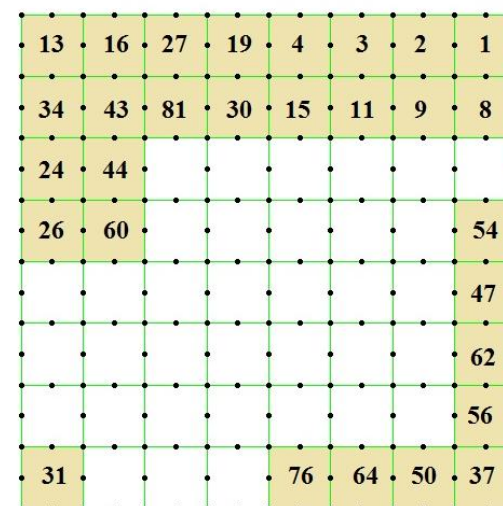
Fig. 7.9 Failure pattern of type: SYAP lamina for boundary condition: CSSC (Contd.)



45° (UPF)



- 45° (UPF)



45° (UPF)

Fig. 7.10 Failure pattern of type: ASCP lamina for boundary condition: SCSC

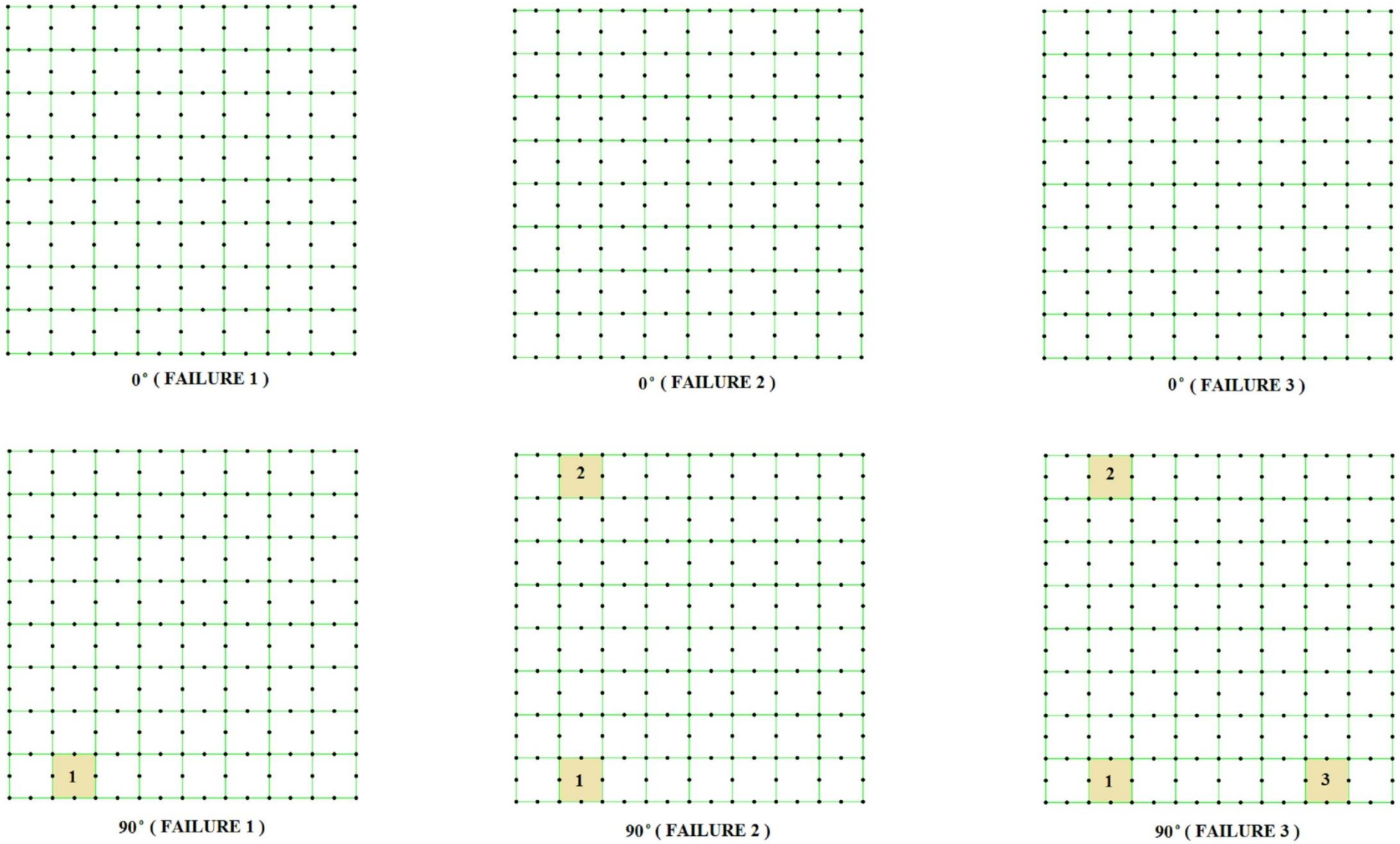
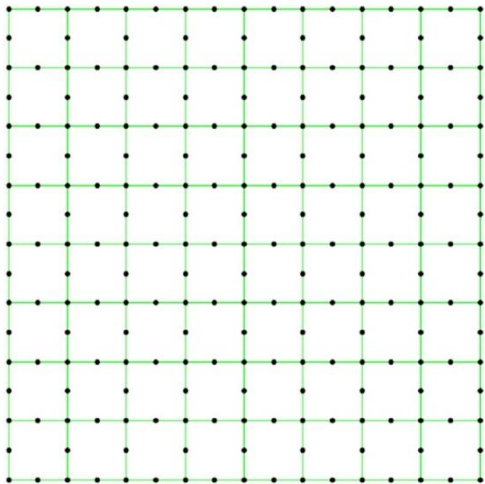
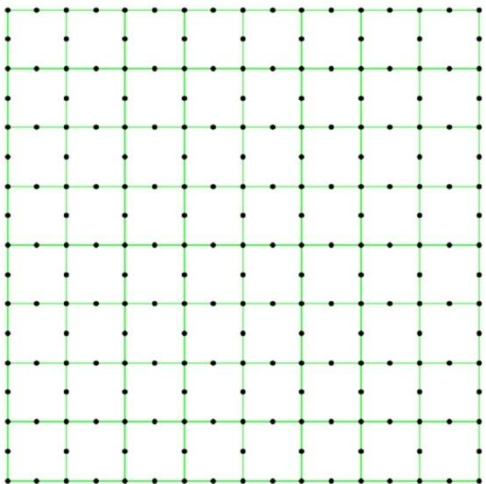


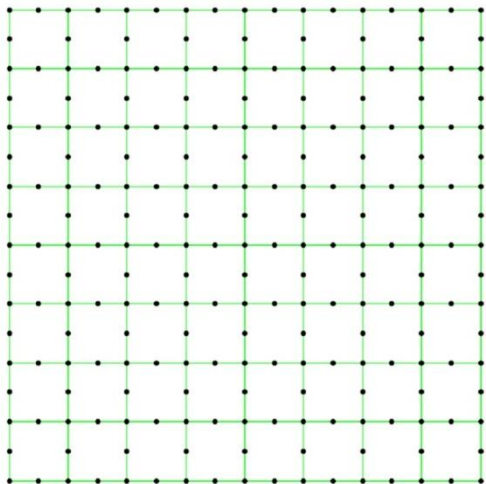
Fig. 7.10 Failure pattern of type: ASCP lamina for boundary condition: SCSC (Contd.)



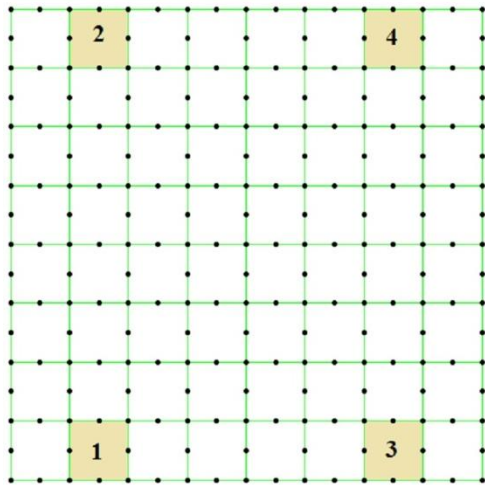
0° (FAILURE 4)



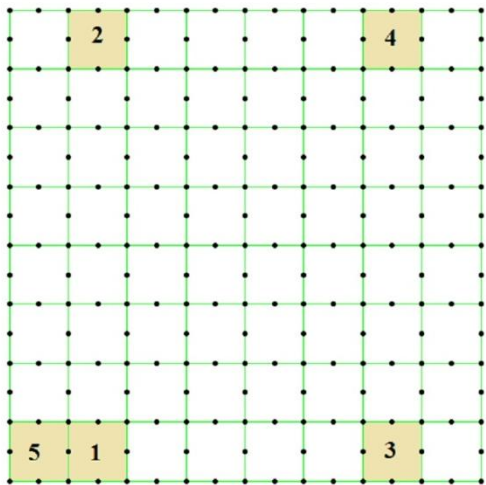
0° (FAILURE 5)



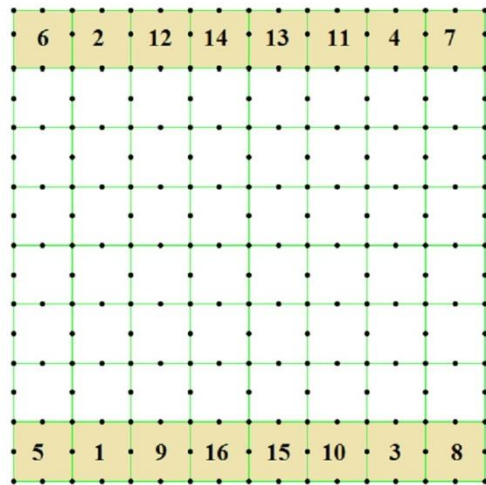
0° (0.25 UPF)



90° (FAILURE 4)

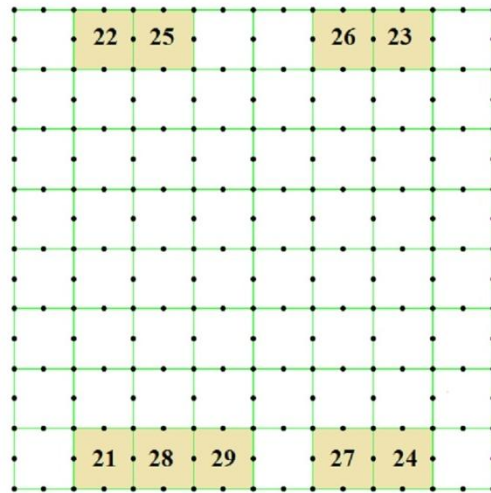


90° (FAILURE 5)

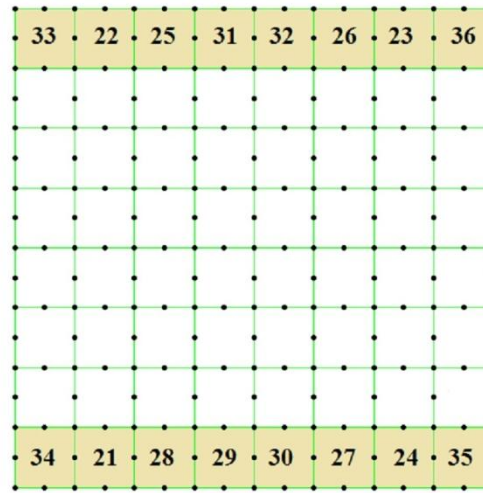


90° (0.25 UPF)

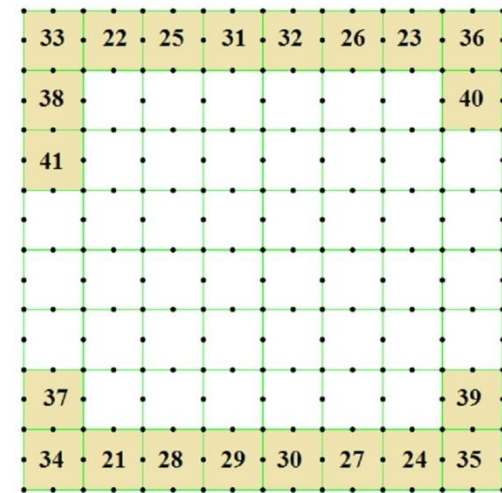
Fig. 7.10 Failure pattern of type: ASCP lamina for boundary condition: SCSC (Contd.)



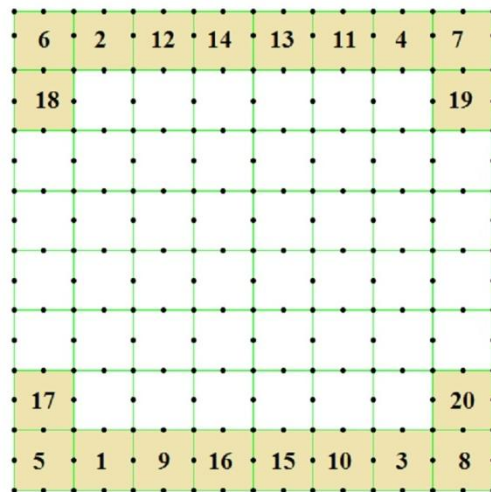
0° (0.5 UPF)



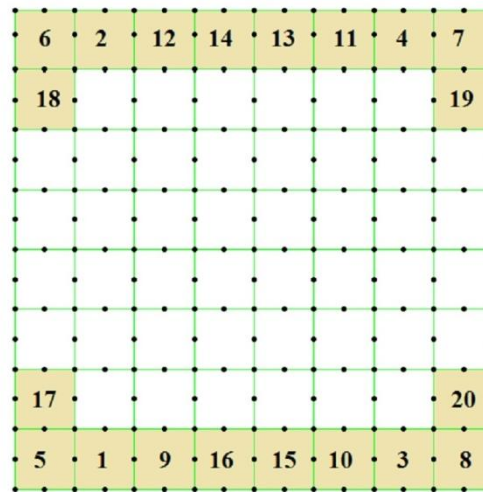
0° (0.75 UPF)



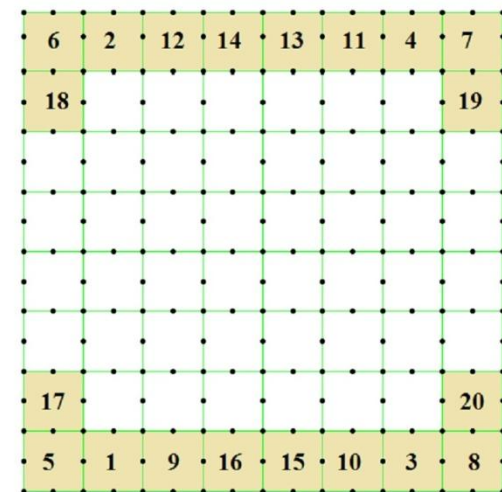
0° (UPF)



90° (0.5 UPF)

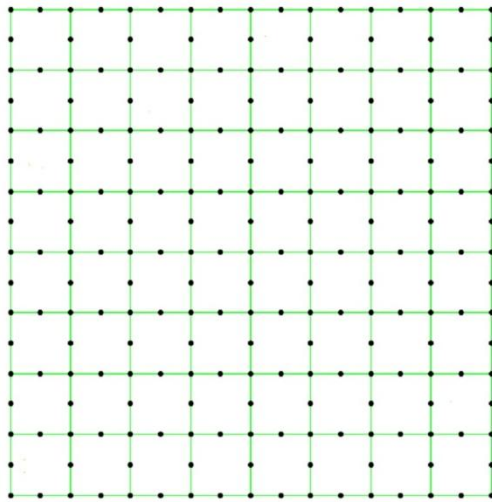


90° (0.75 UPF)

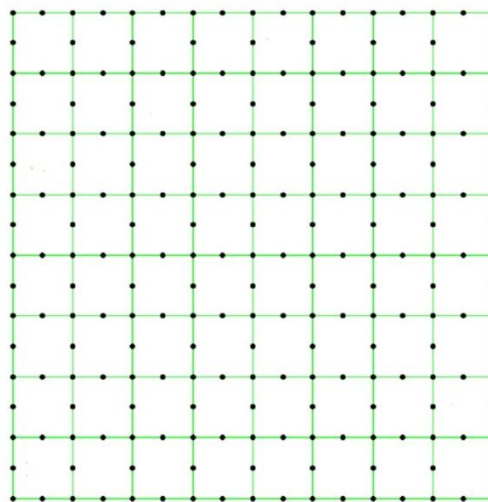


90° (UPF)

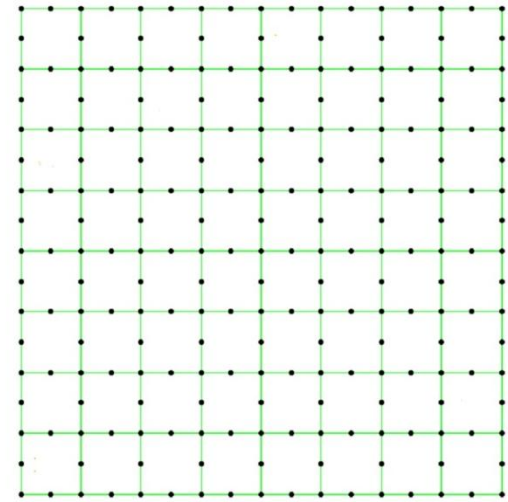
Fig. 7.11 Failure pattern of type: ASAP lamina for boundary condition: SCSC



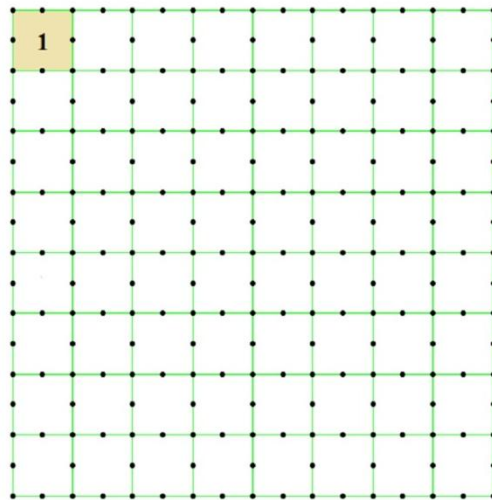
45° (FAILURE 1)



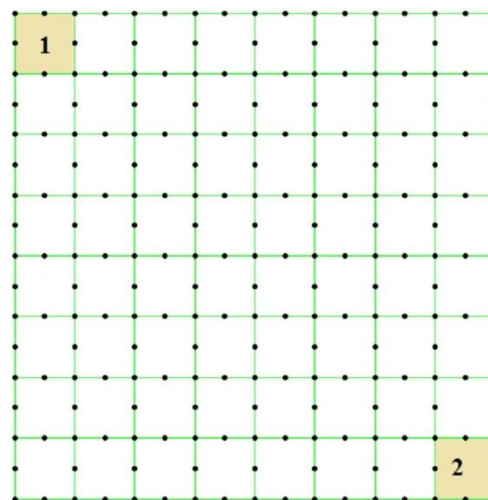
45° (FAILURE 2)



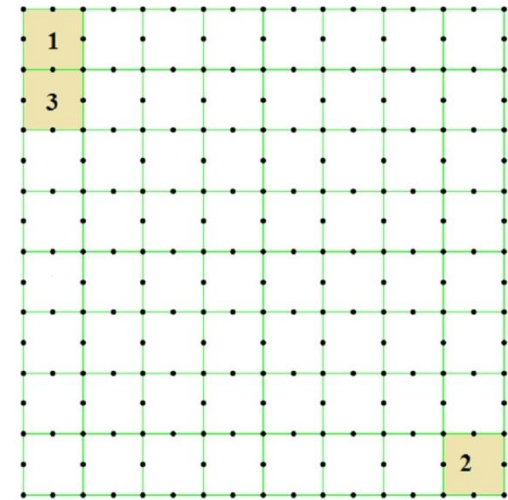
45° (FAILURE 3)



- 45° (FAILURE 1)

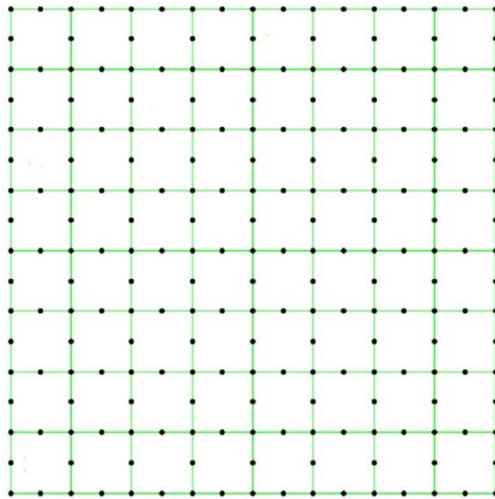


- 45° (FAILURE 2)

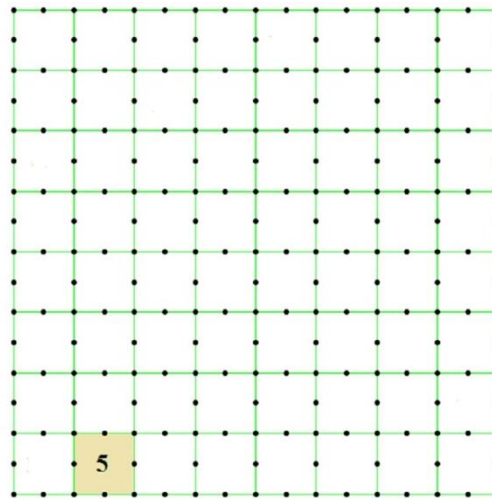


- 45° (FAILURE 3)

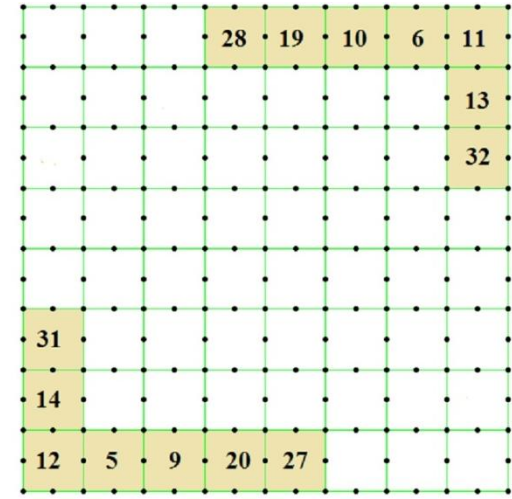
Fig. 7.11 Failure pattern of type: ASAP lamina for boundary condition: SCSC (Contd.)



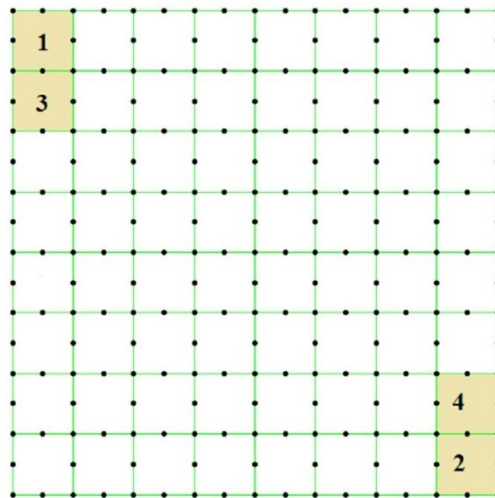
45° (FAILURE 4)



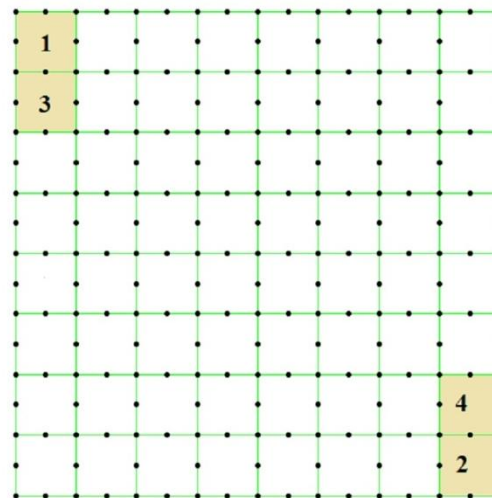
45° (FAILURE 5)



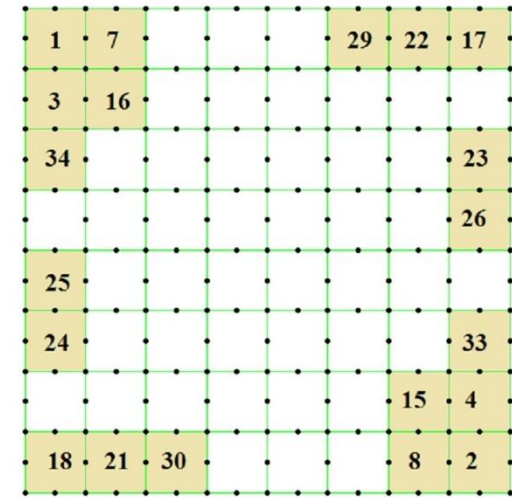
45° (0.25 UPF)



-45° (FAILURE 4)

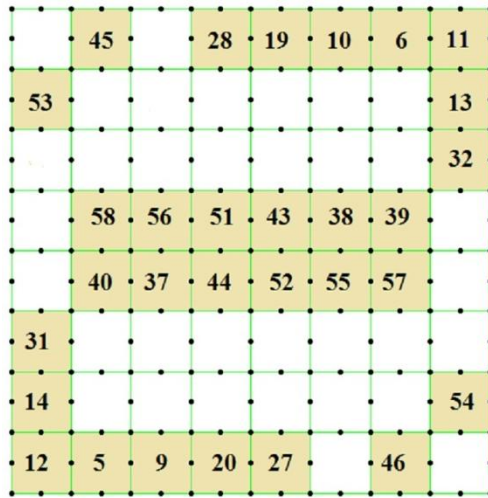


-45° (FAILURE 5)

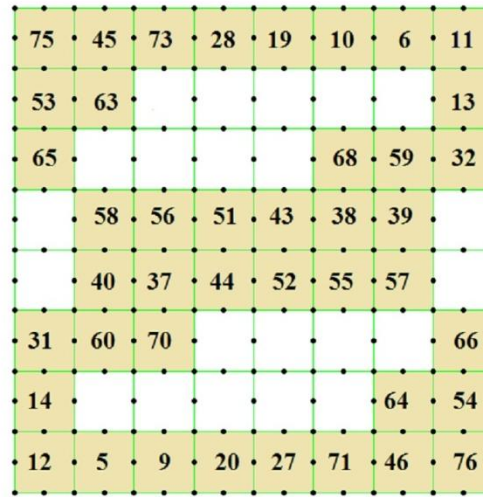


-45° (0.25 UPF)

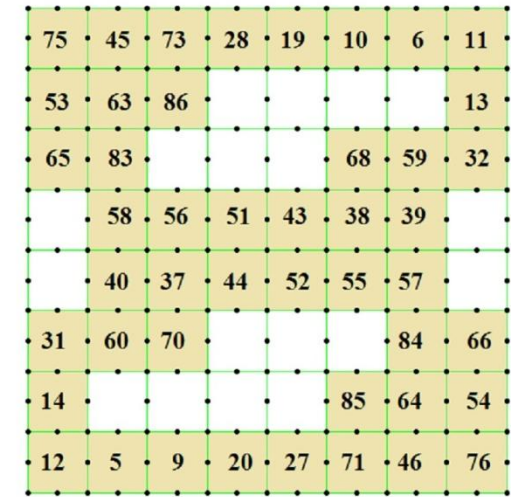
Fig. 7.11 Failure pattern of type: ASAP lamina for boundary condition: SCSC (Contd.)



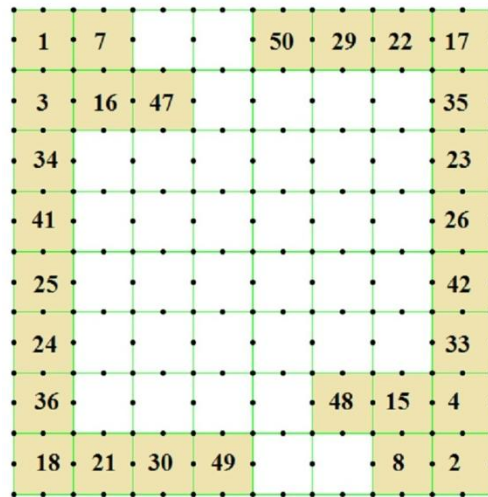
45° (0.5 UPF)



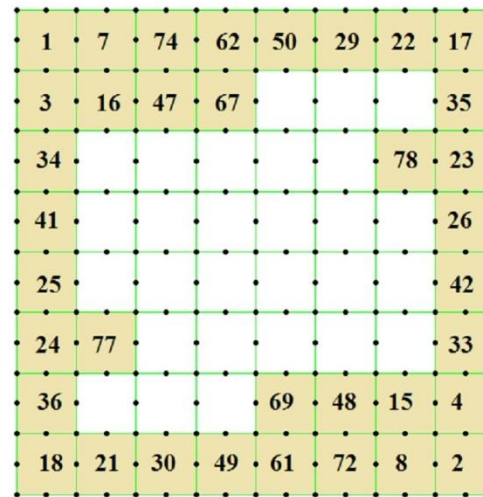
45° (0.75 UPF)



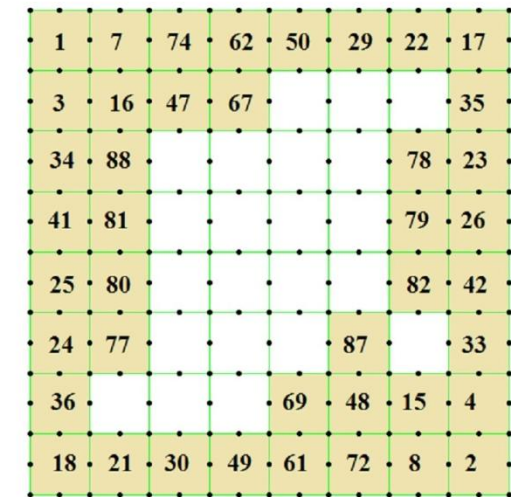
45° (UPF)



-45° (0.5 UPF)

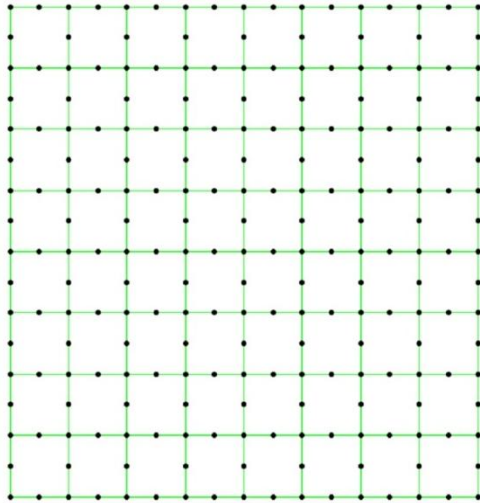


-45° (0.75 UPF)

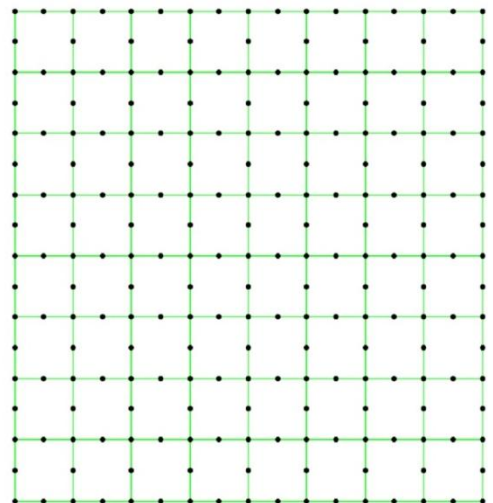


-45° (UPF)

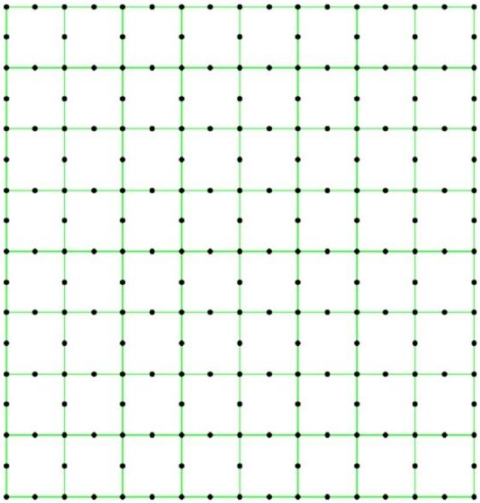
Fig. 7.12 Failure pattern of type: SYCP lamina for boundary condition: SCSC



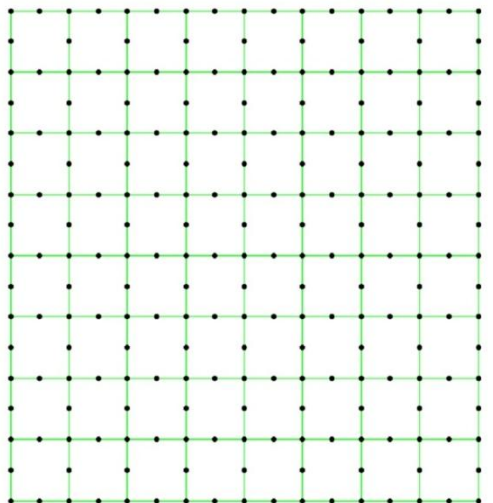
0° (FAILURE 1)



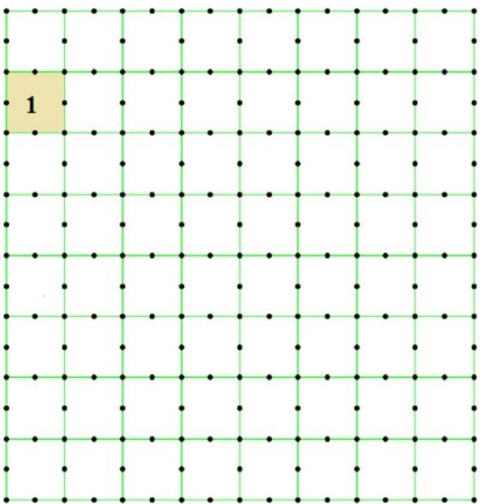
0° (FAILURE 2)



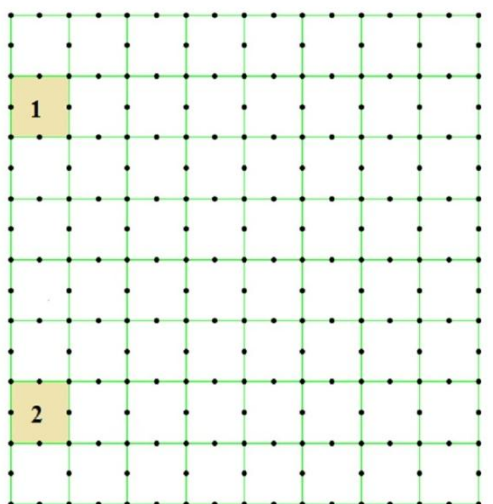
90° (FAILURE 1)



90° (FAILURE 2)

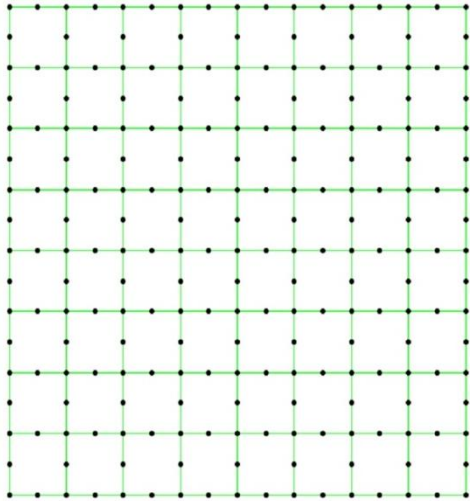


0° (FAILURE 1)

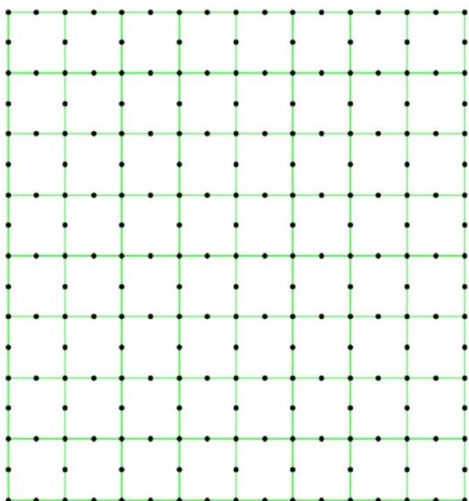


0° (FAILURE 2)

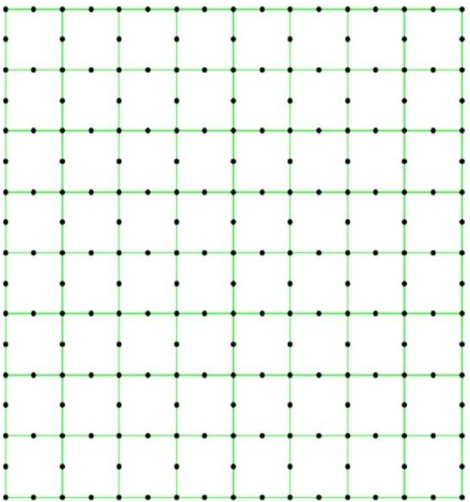
Fig. 7.12 Failure pattern of type: SYCP lamina for boundary condition: SCSC (Contd.)



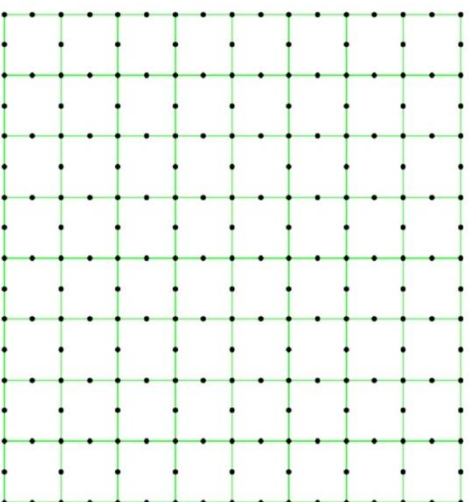
0° (FAILURE 3)



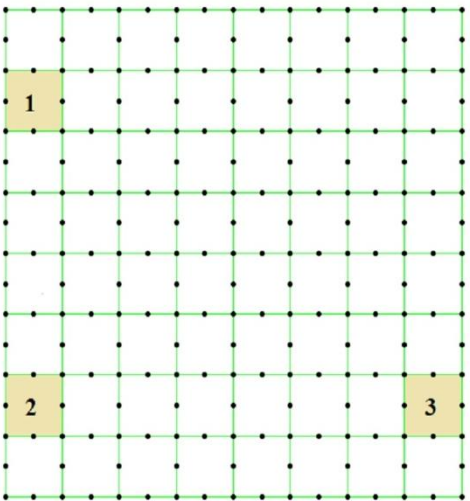
0° (FAILURE 4)



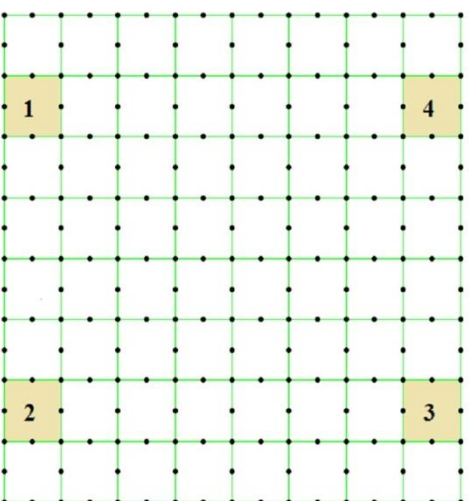
90° (FAILURE 3)



90° (FAILURE 4)



0° (FAILURE 3)



0° (FAILURE 4)

Fig. 7.12 Failure pattern of type: SYCP lamina for boundary condition: SCSC (Contd.)

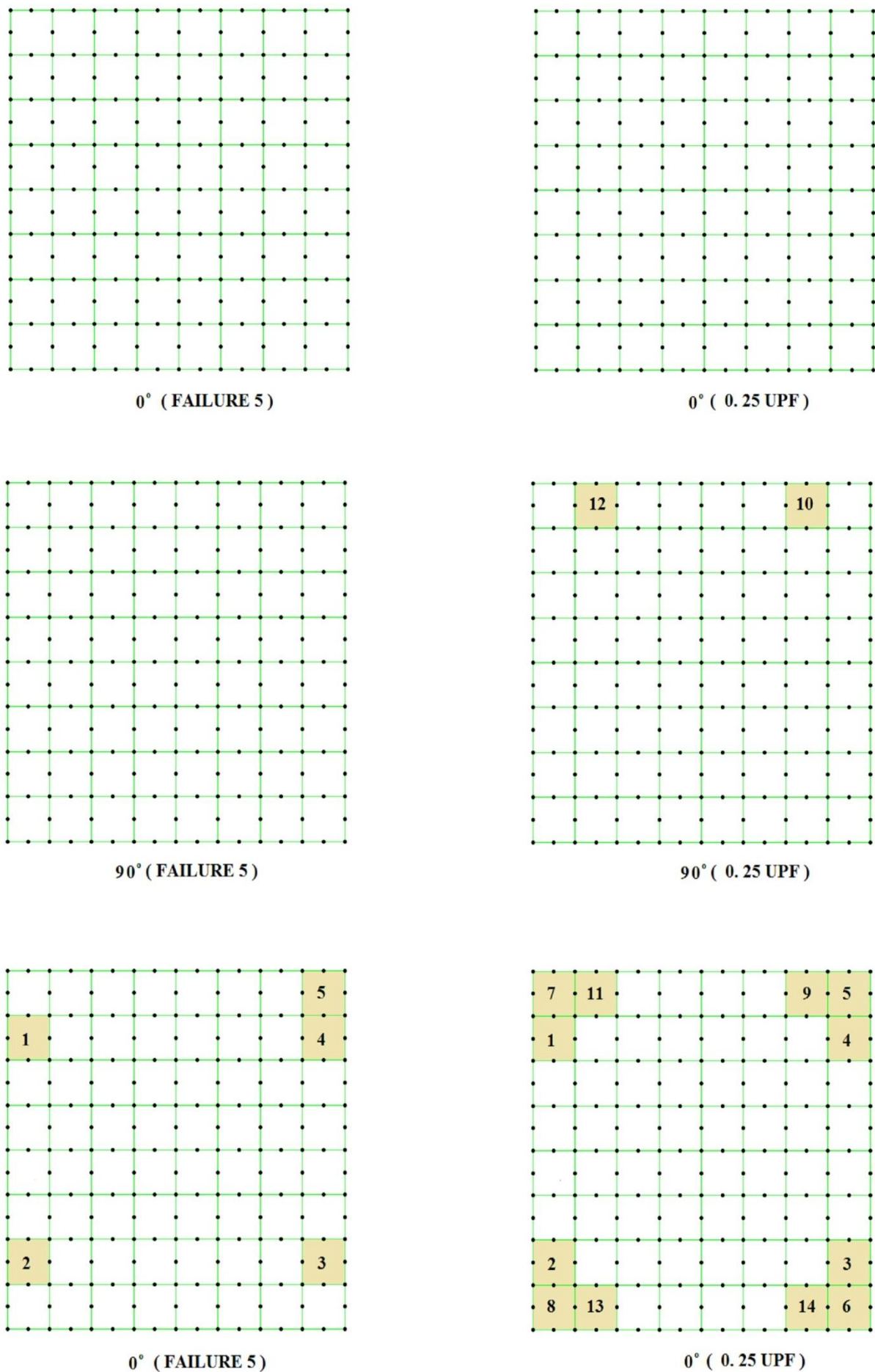
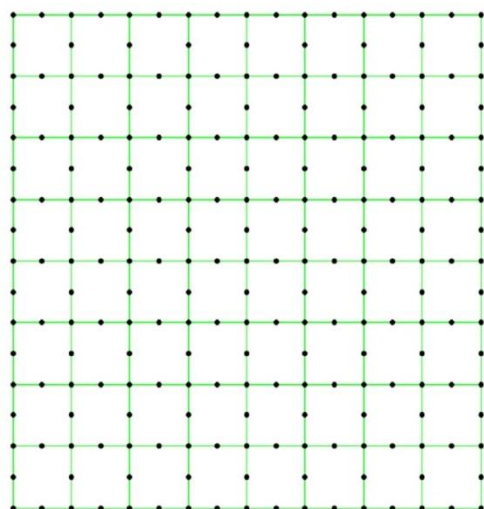
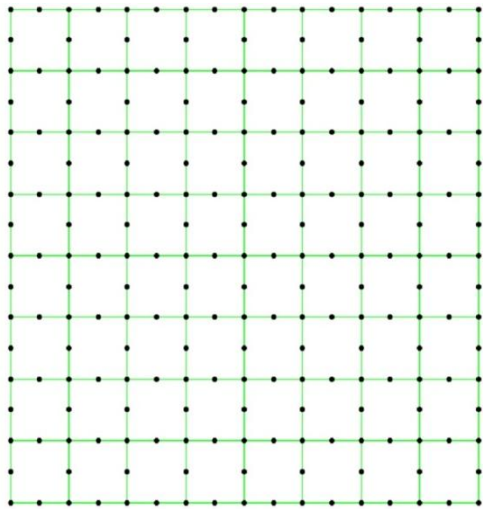


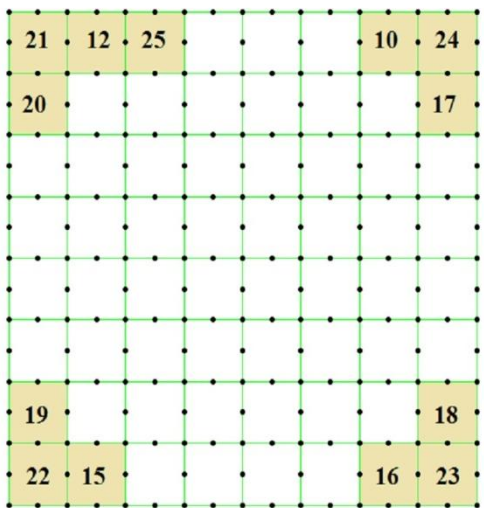
Fig. 7.12 Failure pattern of type: SYCP lamina for boundary condition: SCSC (Contd.)



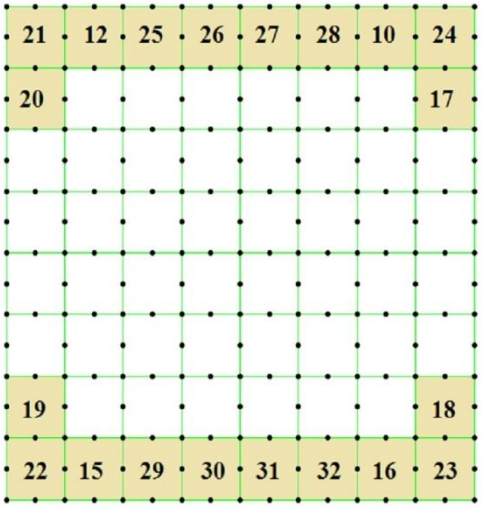
0° (0.5 UPF)



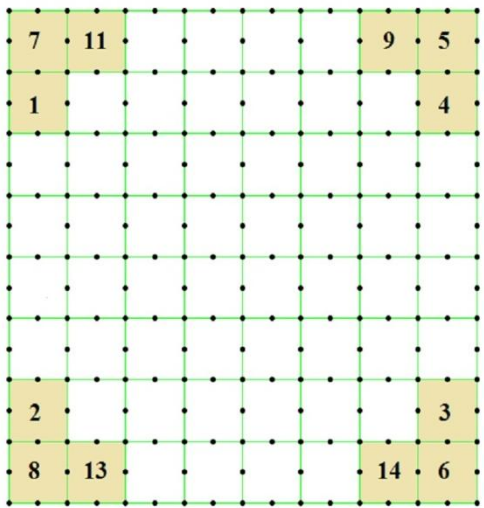
0° (0.75 UPF)



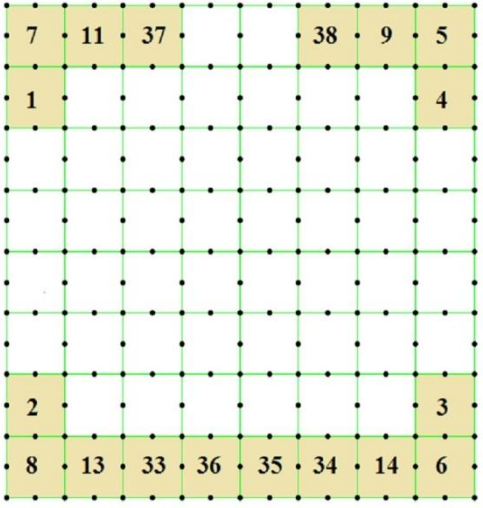
90° (0.5 UPF)



90° (0.75 UPF)



0° (0.5 UPF)



0° (0.75 UPF)

Fig. 7.12 Failure pattern of type: SYCP lamina for boundary condition: SCSC (Contd.)

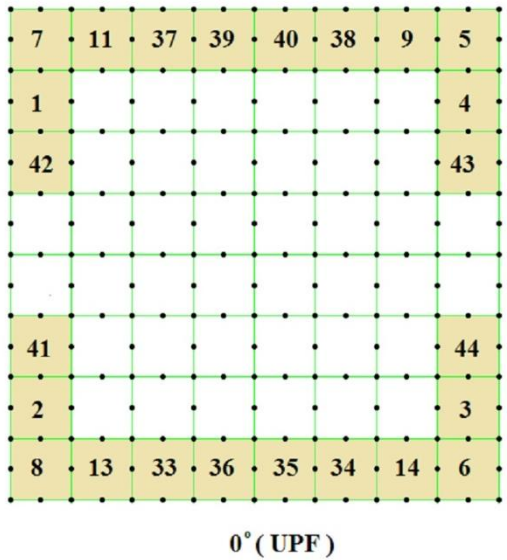
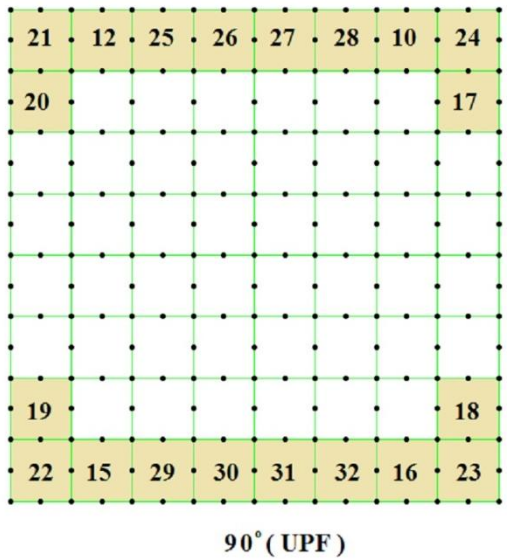
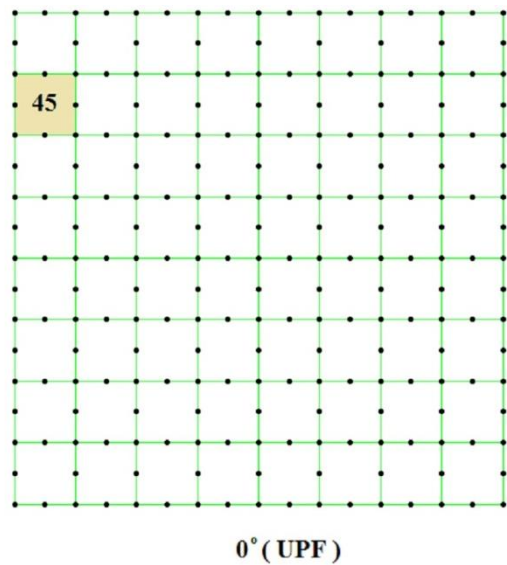
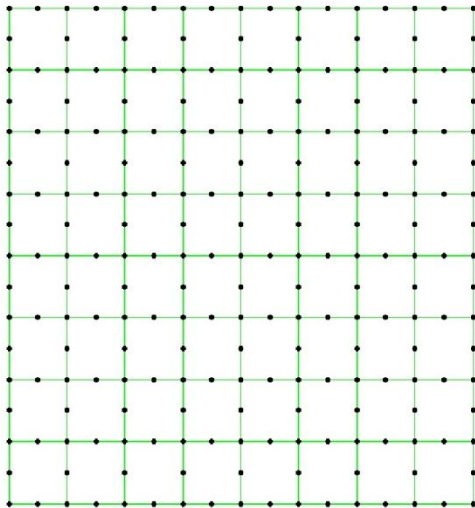
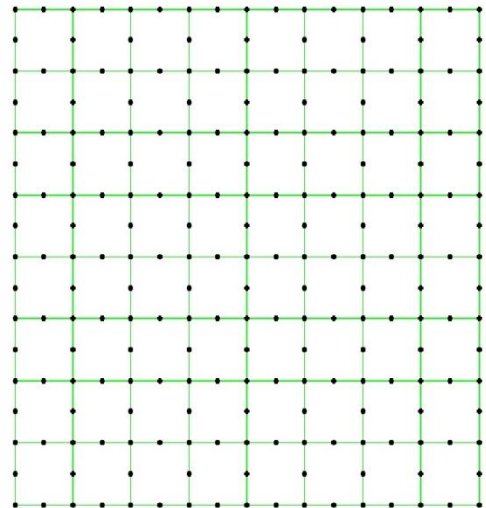


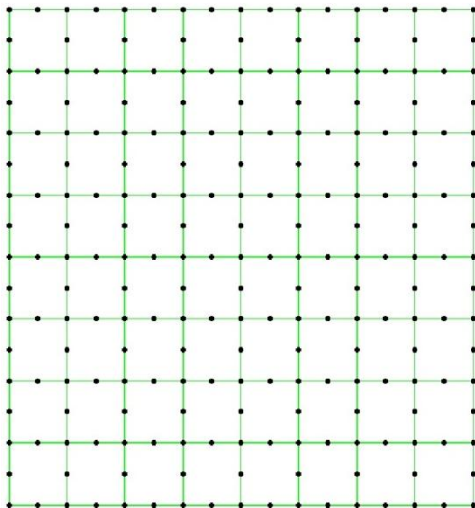
Fig. 7.13 Failure pattern of type: SYAP lamina for boundary condition: SCSC



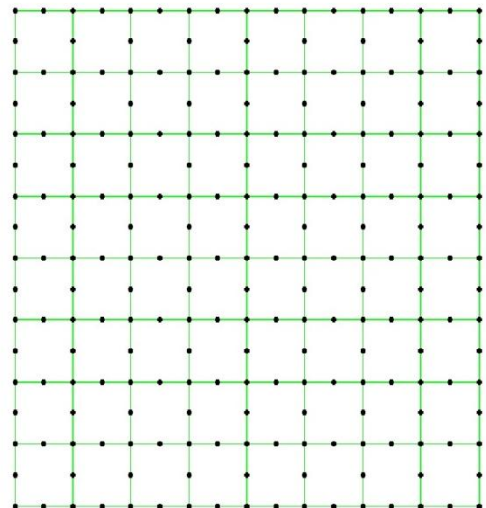
45° (FAILURE 1)



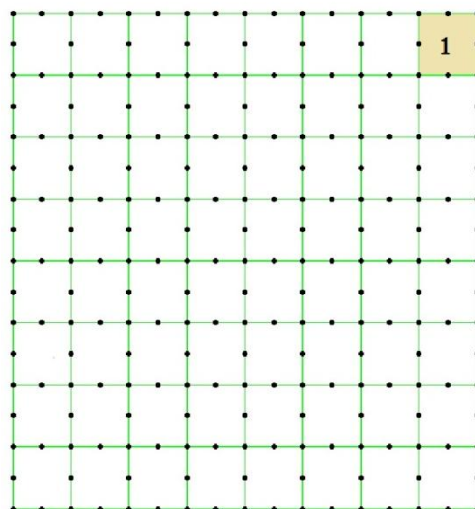
45° (FAILURE 2)



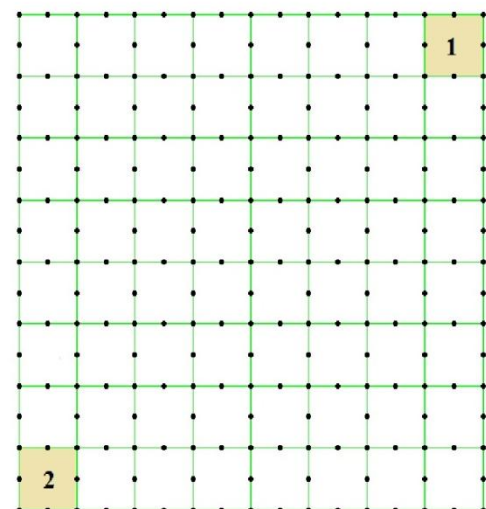
- 45° (FAILURE 1)



- 45° (FAILURE 2)

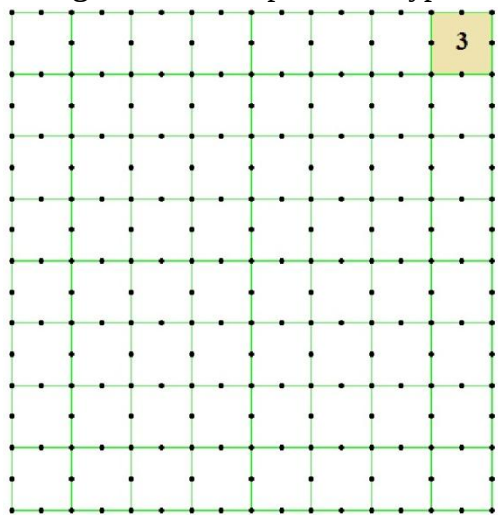


45° (FAILURE 1)

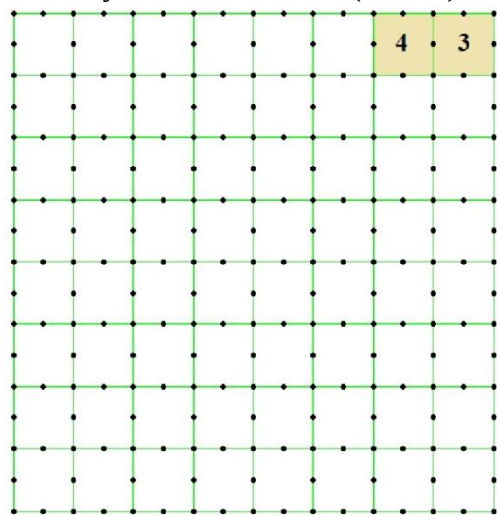


45° (FAILURE 2)

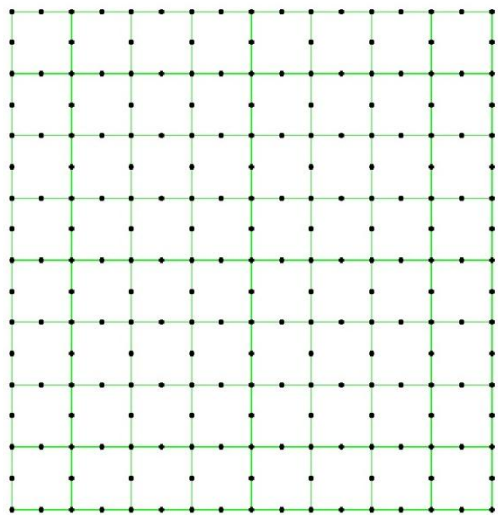
Fig. 7.13 Failure pattern of type: SYAP lamina for boundary condition: SCSC (Contd.)



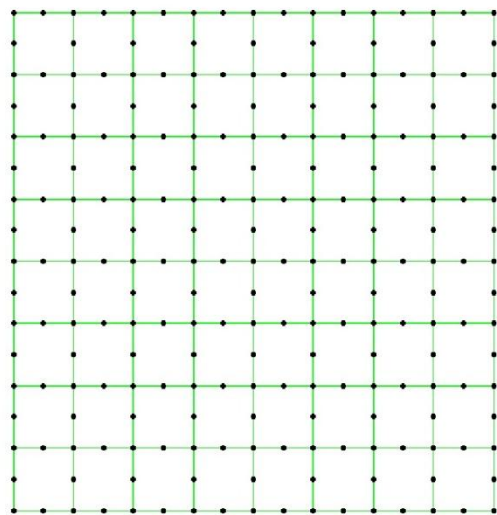
45° (FAILURE 3)



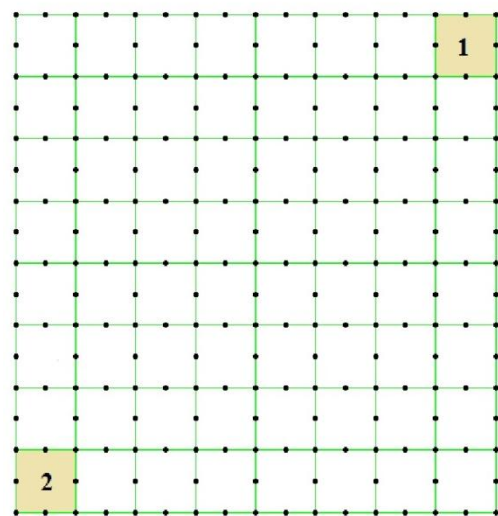
45° (FAILURE 4)



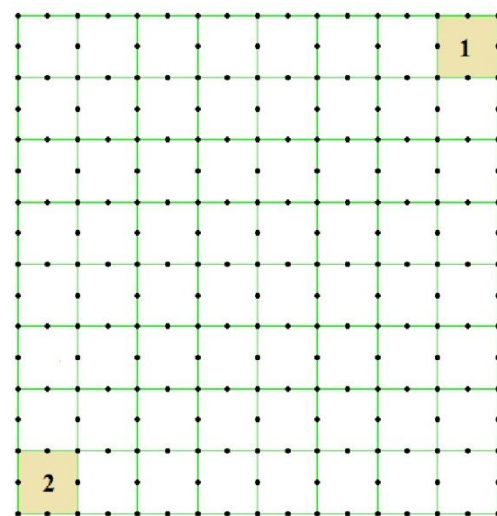
- 45° (FAILURE 3)



- 45° (FAILURE 4)

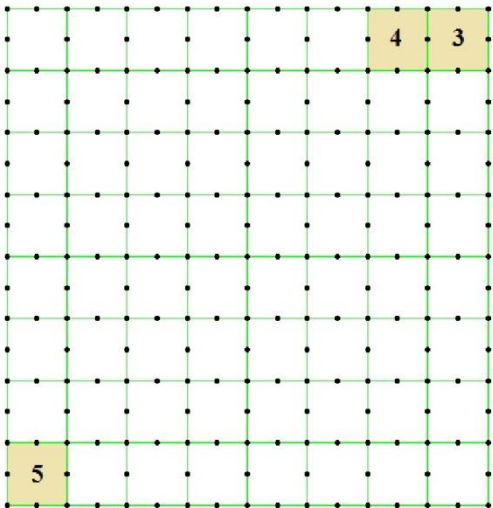


45° (FAILURE 3)

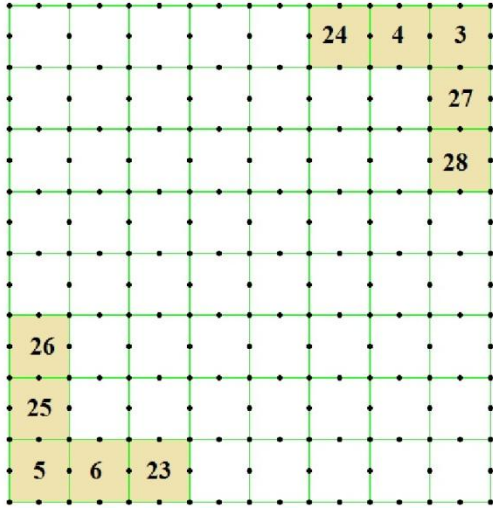


45° (FAILURE 4)

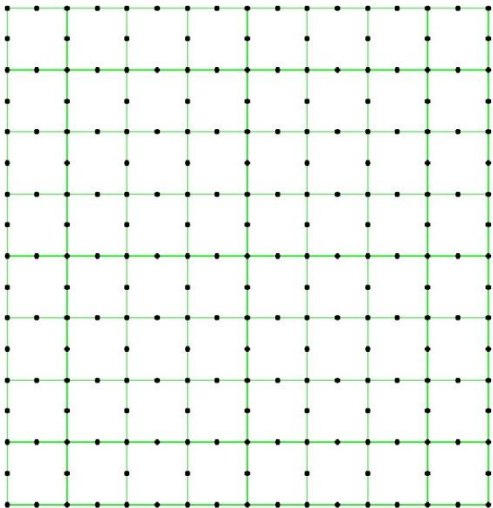
Fig. 7.13 Failure pattern of type: SYAP lamina for boundary condition: SCSC (Contd.)



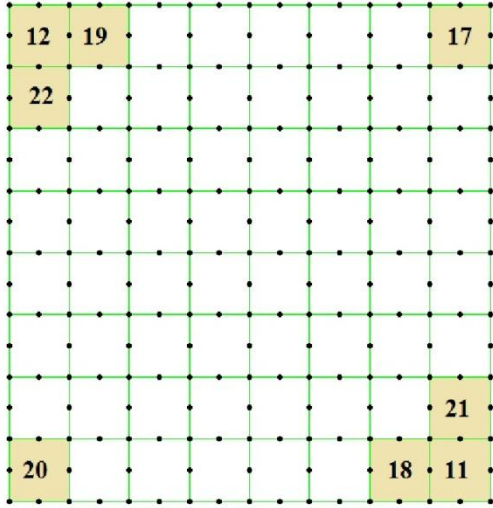
45° (FAILURE 5)



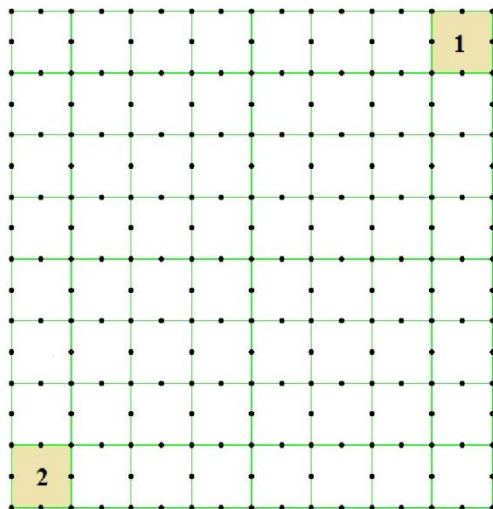
45° (0.25 UPF)



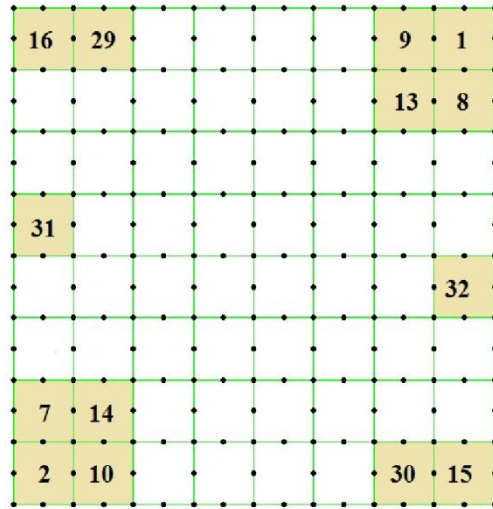
- 45° (FAILURE 5)



- 45° (0.25 UPF)

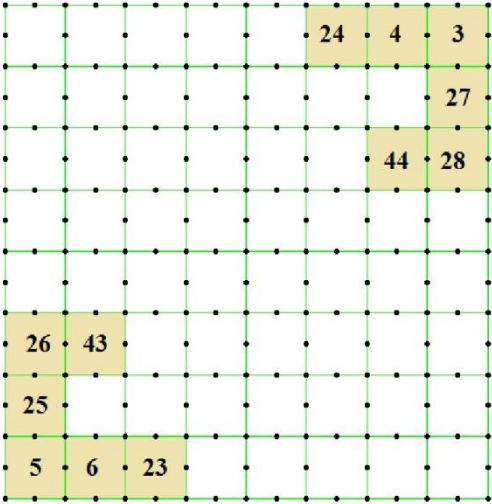


45° (FAILURE 5)

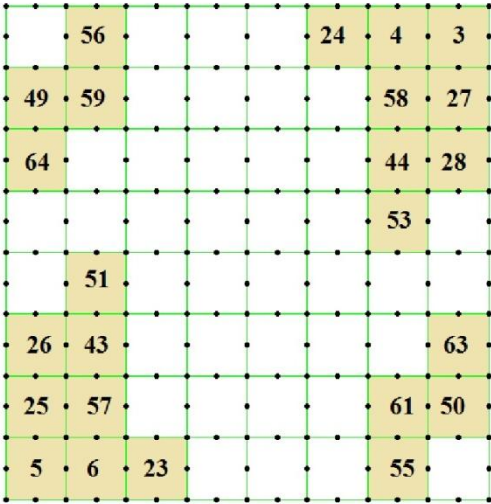


45° (0.25 UPF)

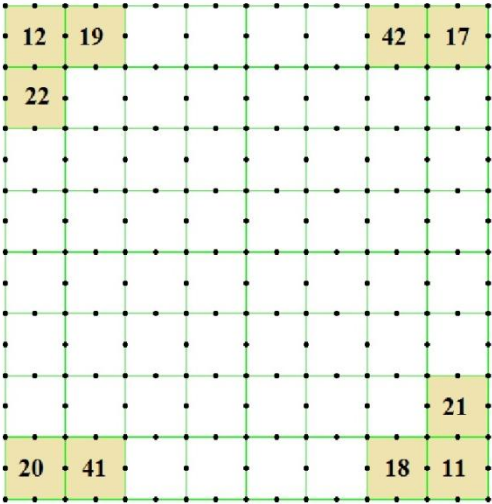
Fig. 7.13 Failure pattern of type: SYAP lamina for boundary condition: SCSC (Contd.)



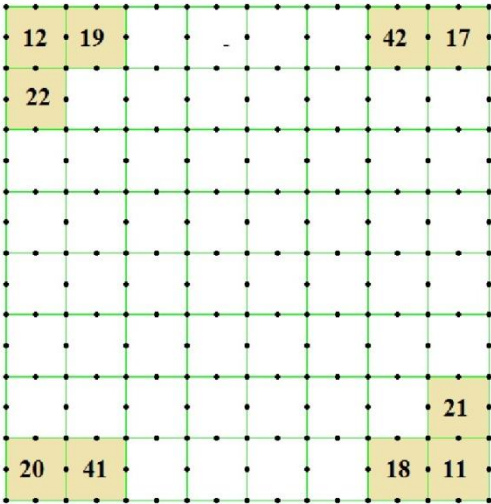
45° (0.5 UPF)



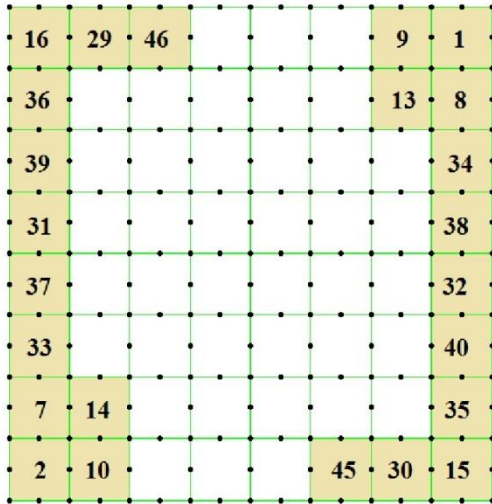
45° (0.75 UPF)



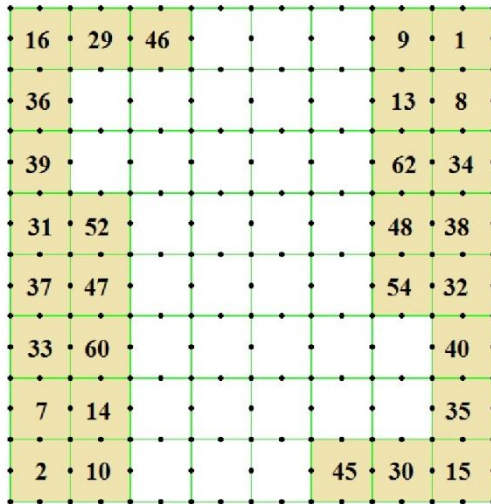
- 45° (0.5 UPF)



- 45° (0.75 UPF)

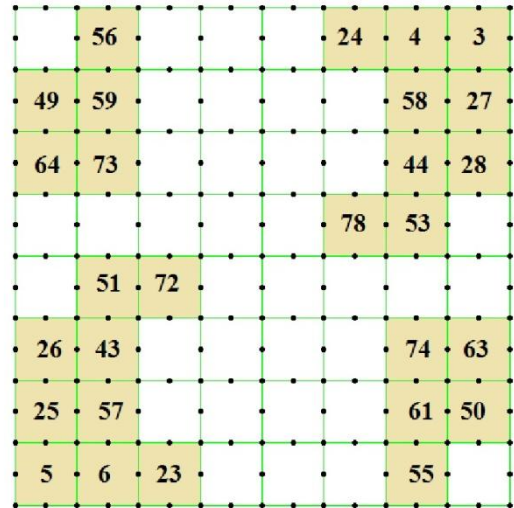


45° (0.5 UPF)

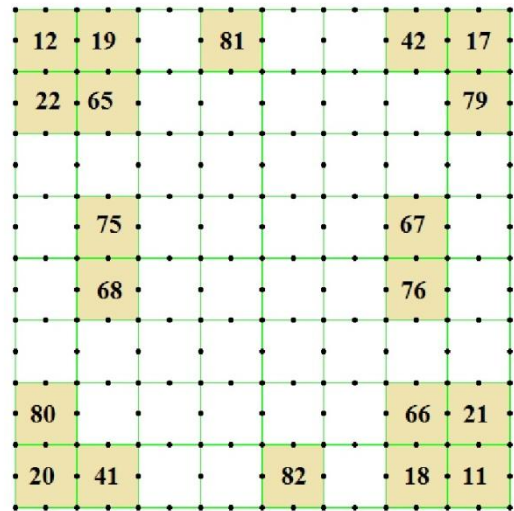


45° (0.75 UPF)

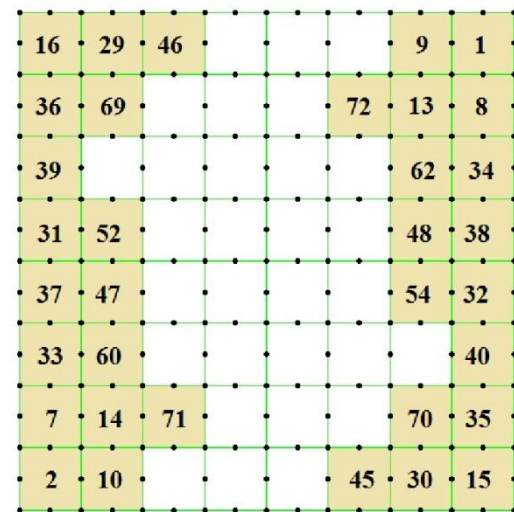
Fig. 7.13 Failure pattern of type: SYAP lamina for boundary condition: SCSC (Contd.)



45°(UPF)



- 45°(UPF)



45°(UPF)

Fig. 7.14 Failure pattern of type: ASCP lamina for boundary condition: SSSS

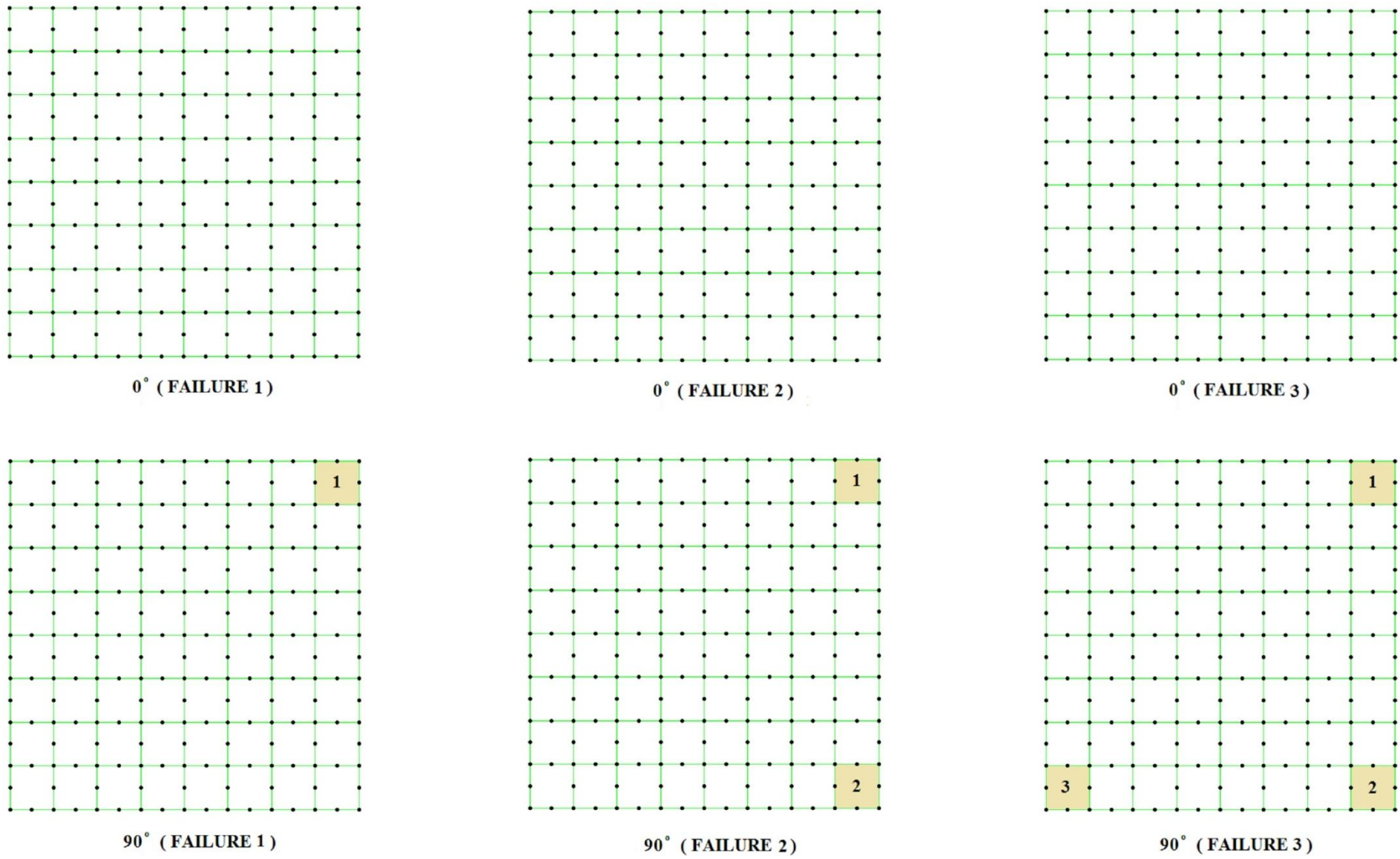
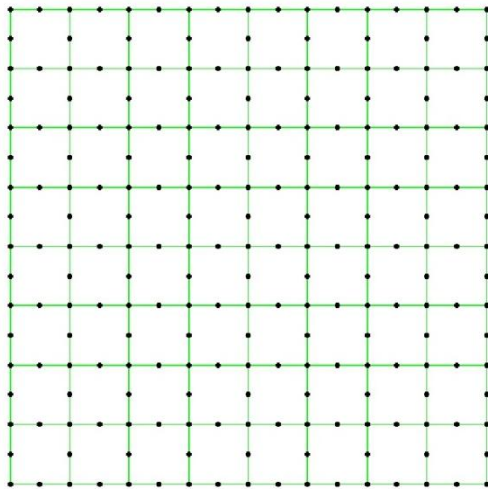
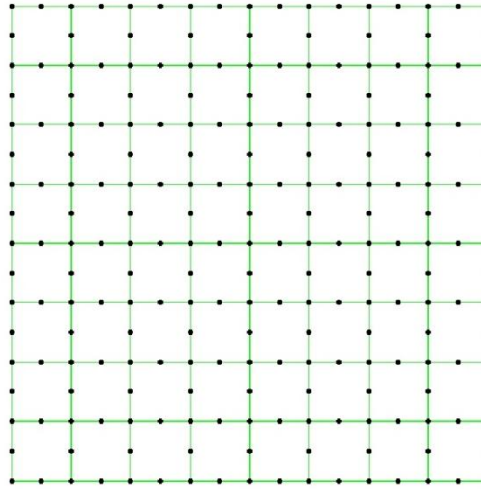


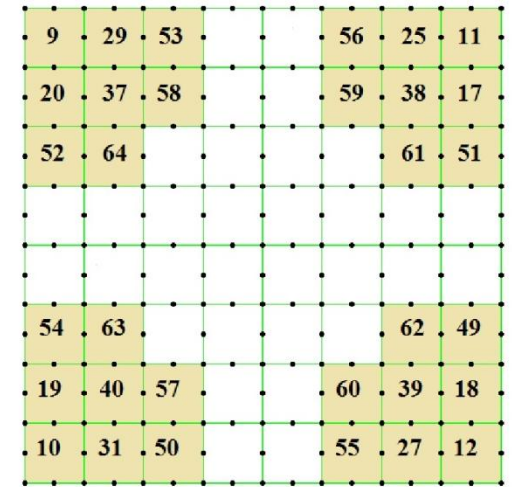
Fig. 7.14 Failure pattern of type: ASCP lamina for boundary condition: SSSS (Contd.)



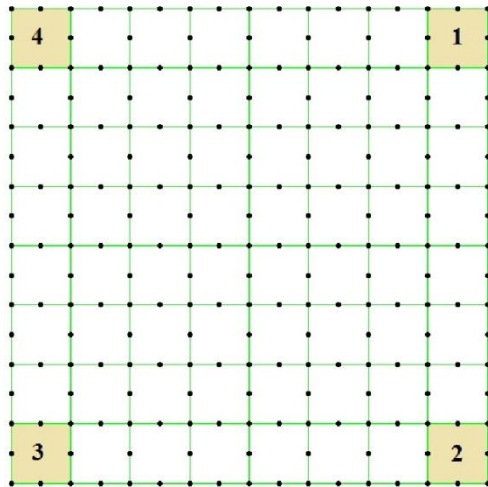
0° (FAILURE 4)



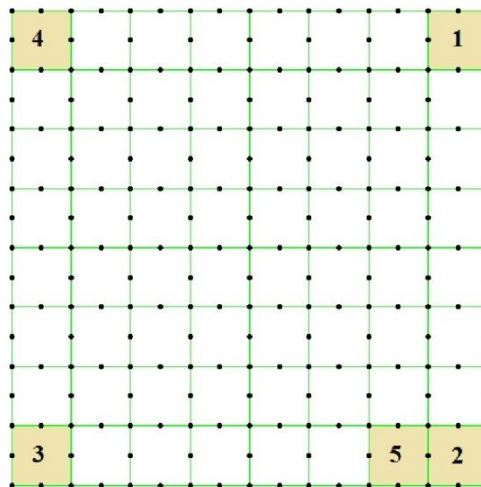
0° (FAILURE 5)



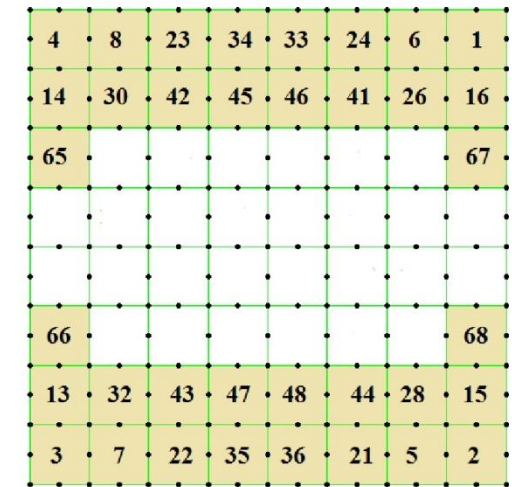
0° (0.25 UPF)



90° (FAILURE 4)

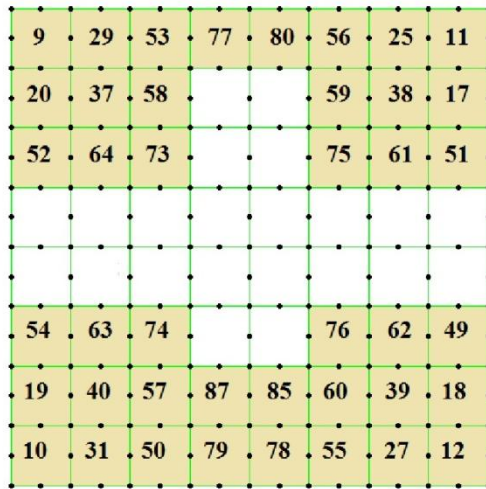


90° (FAILURE 5)

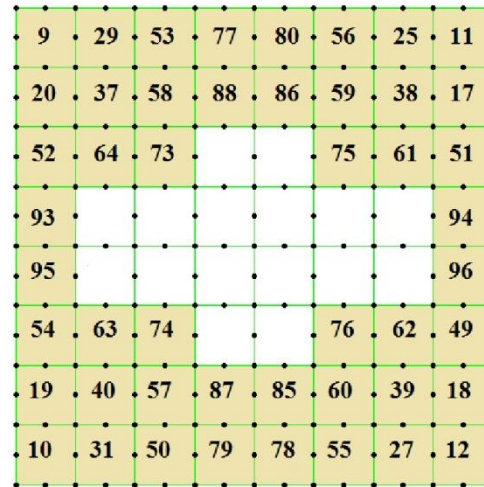


90° (0.25 UPF)

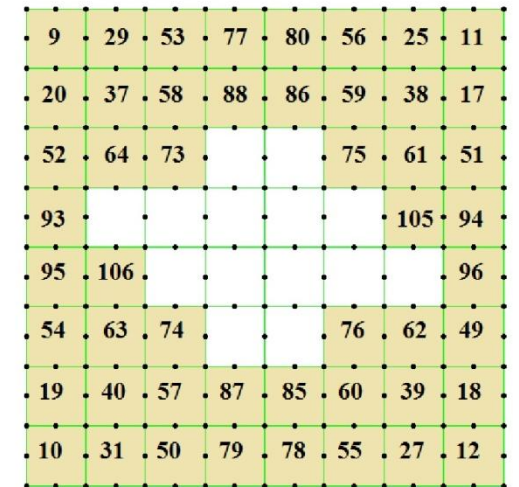
Fig. 7.14 Failure pattern of type: ASCP lamina for boundary condition: SSSS (Contd.)



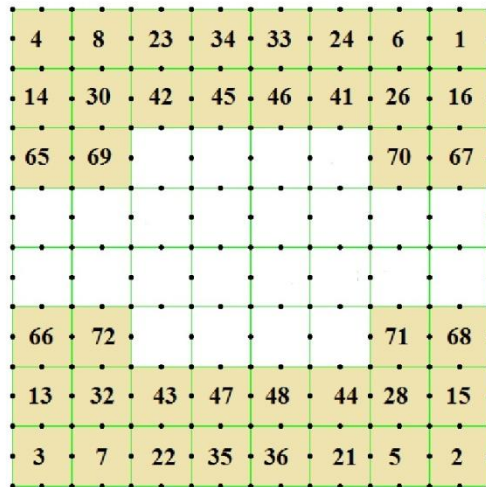
0° (0.5 UPF)



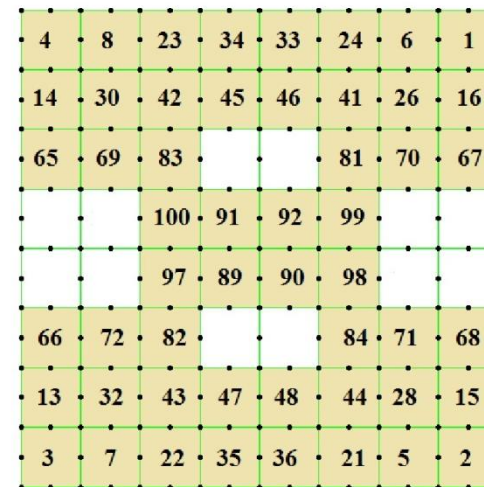
0° (0.75 UPF)



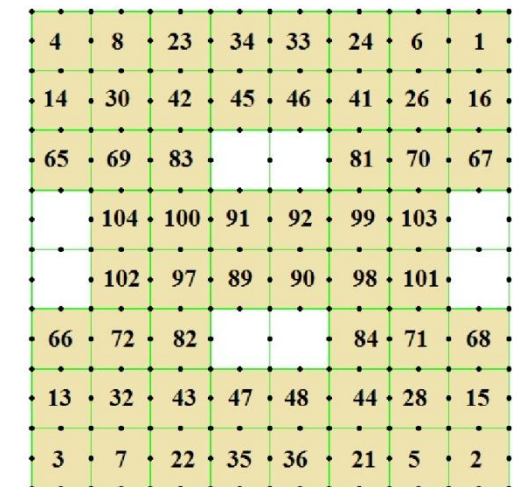
0° (UPF)



90° (0.5 UPF)

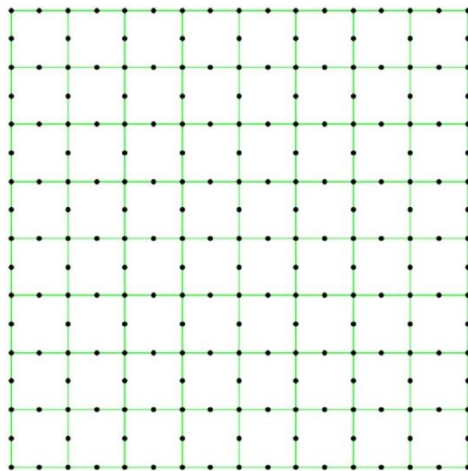


90° (0.75 UPF)

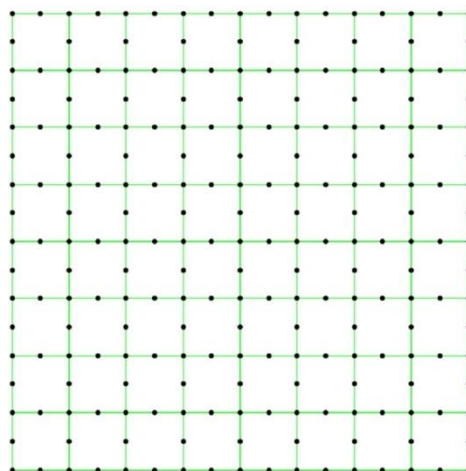


90° (UPF)

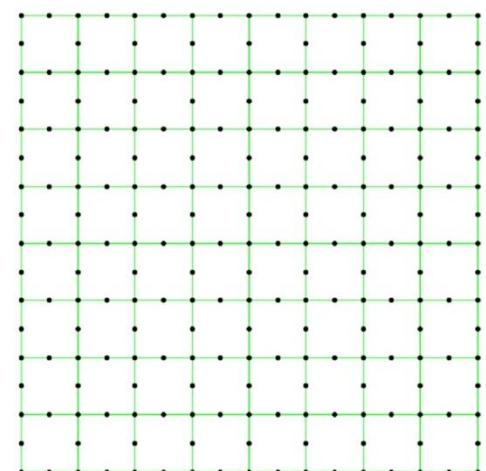
Fig. 7.15 Failure pattern of type: ASAP lamina for boundary condition: SSSS



45° (FAILURE 1)



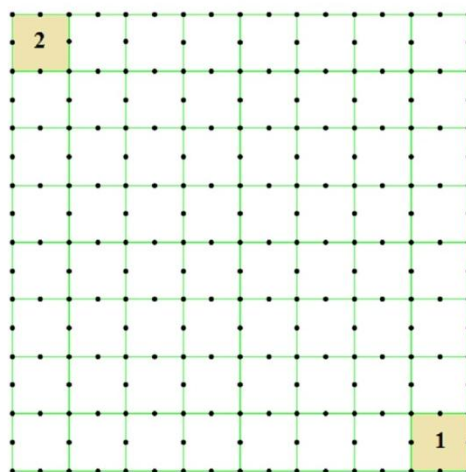
45° (FAILURE 2)



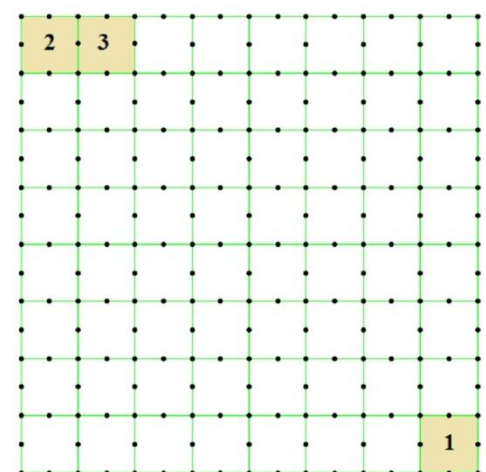
45° (FAILURE 3)



- 45° (FAILURE 1)

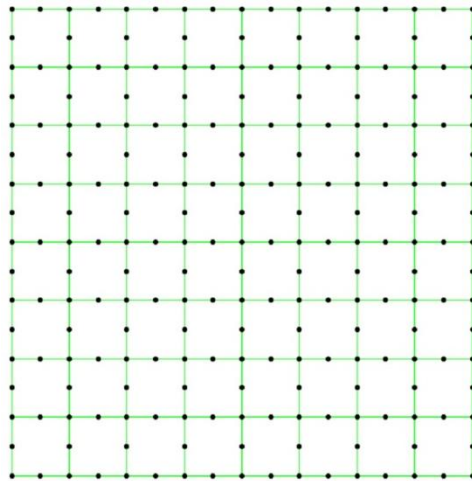


- 45° (FAILURE 2)

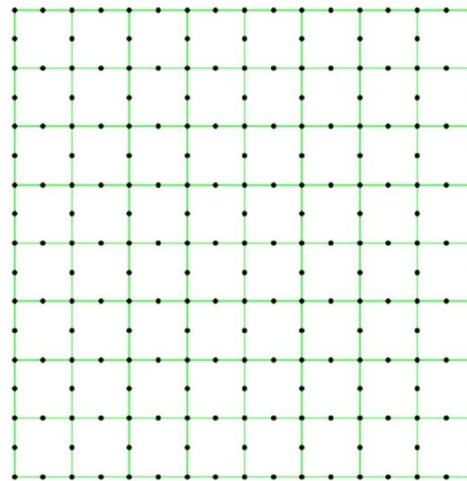


- 45° (FAILURE 3)

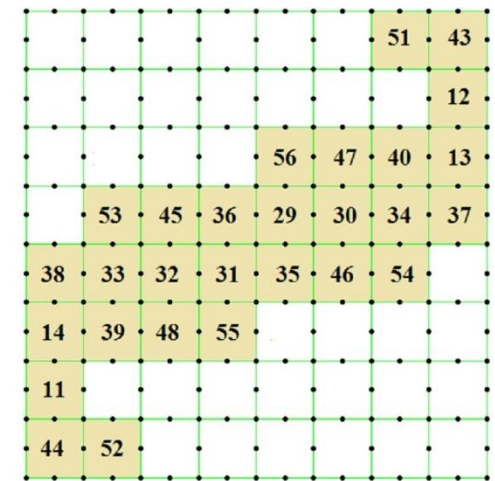
Fig. 7.15 Failure pattern of type: ASAP lamina for boundary condition: SSSS (Contd.)



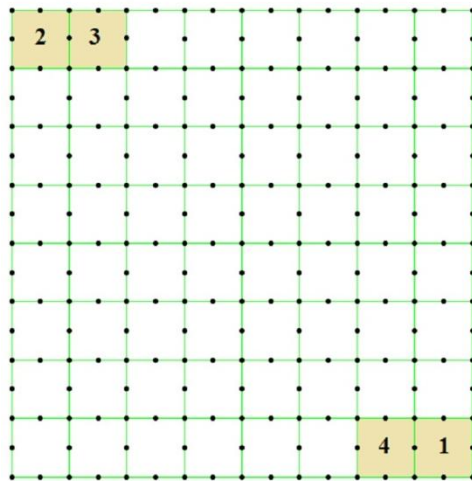
45° (FAILURE 4)



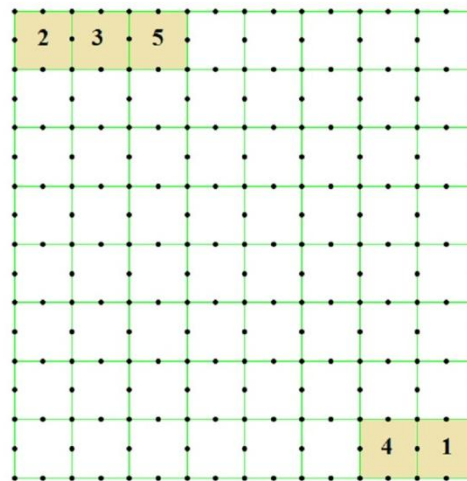
45° (FAILURE 5)



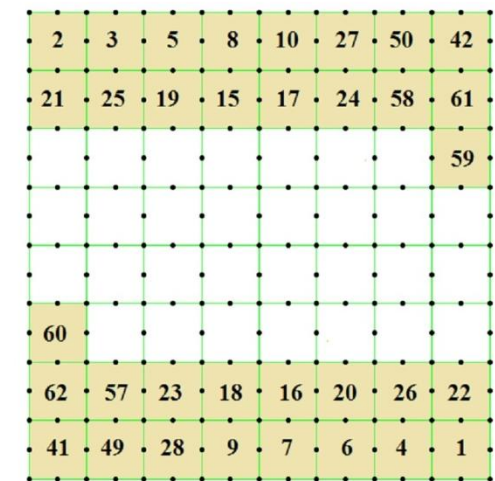
45° (0.25 UPF)



- 45° (FAILURE 4)



- 45° (FAILURE 5)



- 45° (0.25 UPF)

Fig. 7.15 Failure pattern of type: ASAP lamina for boundary condition: SSSS (Contd.)

82	72	70	75	78		51	43
65	64	73	80				12
				56	47	40	13
	53	45	36	29	30	34	37
38	33	32	31	35	46	54	
14	39	48	55				
11				79	74	63	66
44	52		77	76	69	71	81

45° (0.5 UPF)

82	72	70	75	78	94	51	43
65	64	73	80	92		101	12
	105			56	47	40	13
89	53	45	36	29	30	34	37
38	33	32	31	35	46	54	
14	39	48	55				90
11	102		91	79	74	63	66
44	52	93	77	76	69	71	81

45° (0.75 UPF)

82	72	70	75	78	94	51	43
65	64	73	80	92	112	101	12
	105		110	56	47	40	13
89	53	45	36	29	30	34	37
38	33	32	31	35	46	54	
14	39	48	55	109		106	90
11	102	111	91	79	74	63	66
44	52	93	77	76	69	71	81

45° (UPF)

2	3	5	8	10	27	50	42
21	25	19	15	17	24	58	61
						68	59
		88	84	85			
			86	83	87		
60	67						
62	57	23	18	16	20	26	22
41	49	28	9	7	6	4	1

- 45° (0.5 UPF)

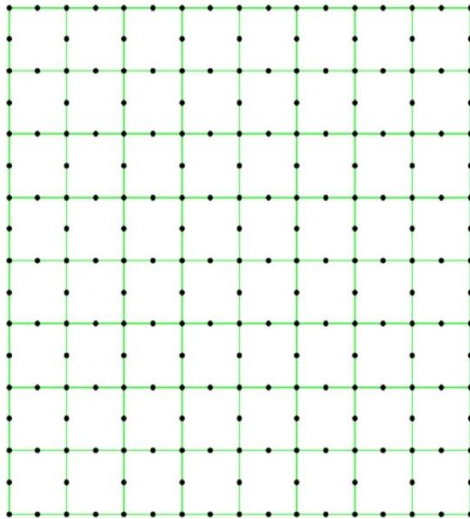
2	3	5	8	10	27	50	42
21	25	19	15	17	24	58	61
	100	103				68	59
	95	88	84	85	98		
		97	86	83	87	96	
60	67				104	99	
62	57	23	18	16	20	26	22
41	49	28	9	7	6	4	1

- 45° (0.75 UPF)

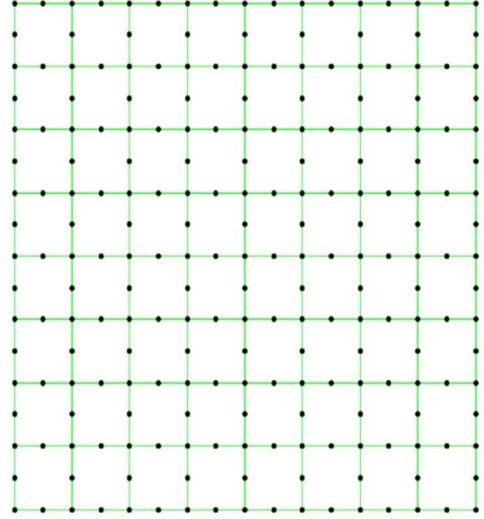
2	3	5	8	10	27	50	42
21	25	19	15	17	24	58	61
	100	103	115		113	68	59
	95	88	84	85	98	118	107
108	117	97	86	83	87	96	
60	67	114		116	104	99	
62	57	23	18	16	20	26	22
41	49	28	9	7	6	4	1

- 45° (UPF)

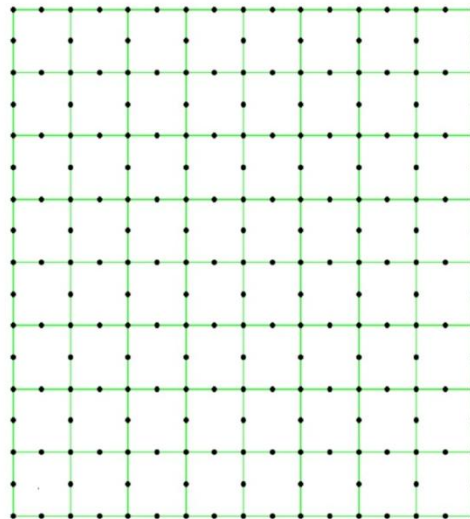
Fig. 7.16 Failure pattern of type: SYCP lamina for boundary condition: SSSS



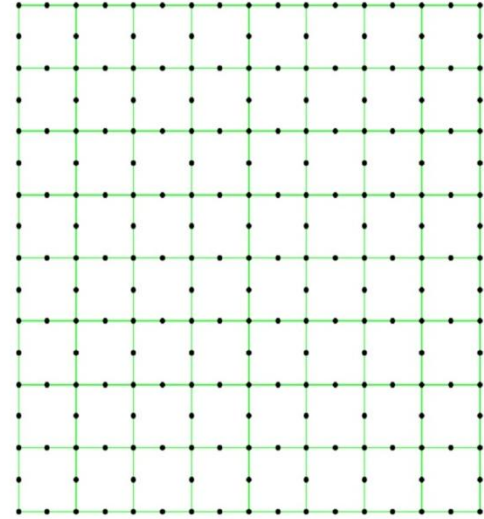
0° (FAILURE 1)



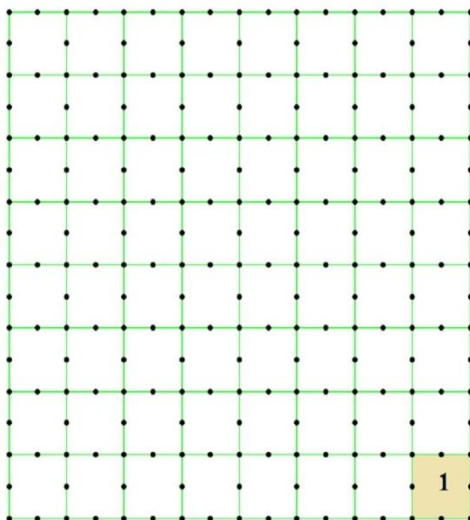
0° (FAILURE 2)



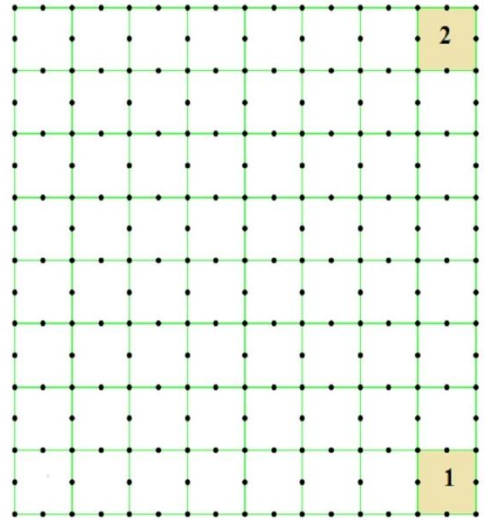
90° (FAILURE 1)



90° (FAILURE 2)

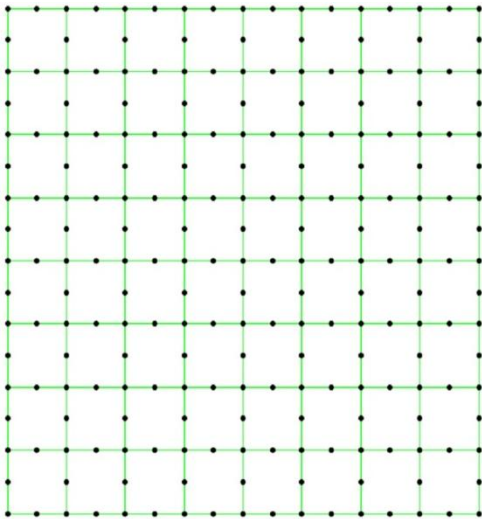


0° (FAILURE 1)

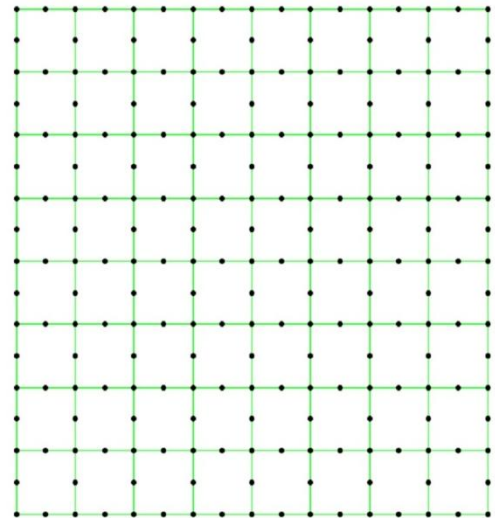


0° (FAILURE 2)

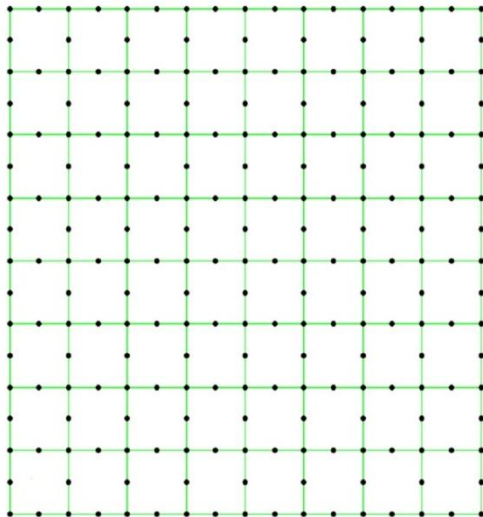
Fig. 7.16 Failure pattern of type: SYCP lamina for boundary condition: SSSS (Contd.)



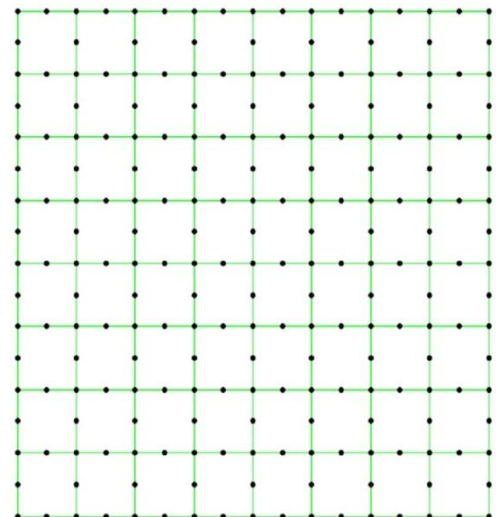
0° (FAILURE 3)



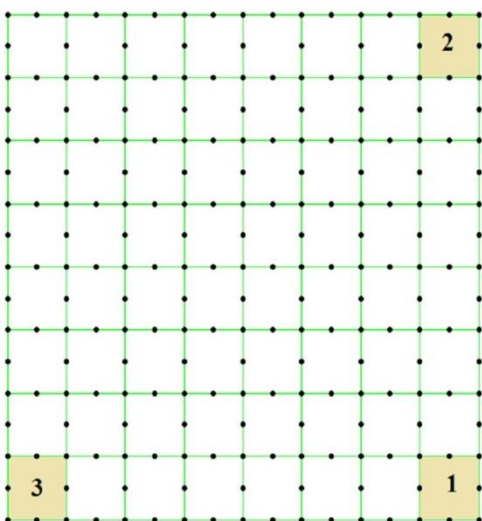
0° (FAILURE 4)



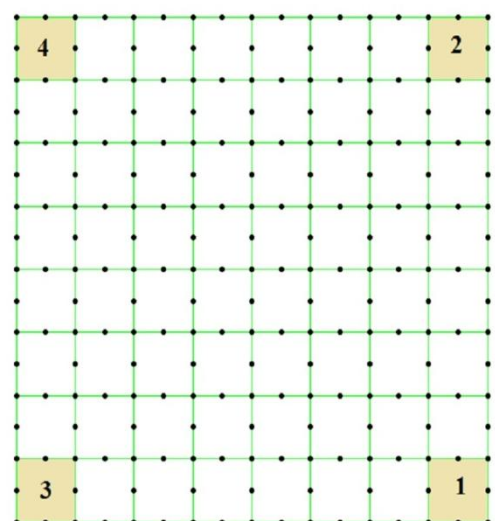
90° (FAILURE 3)



90° (FAILURE 4)

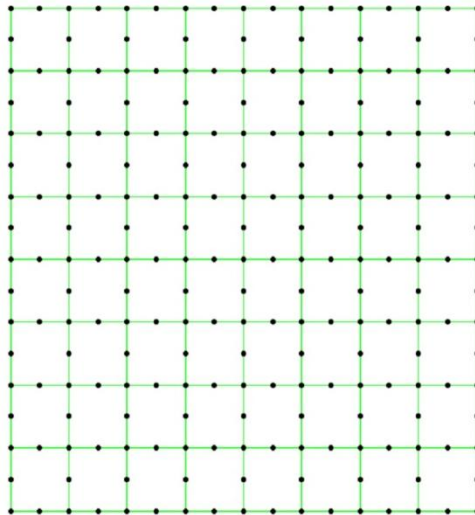


0° (FAILURE 3)

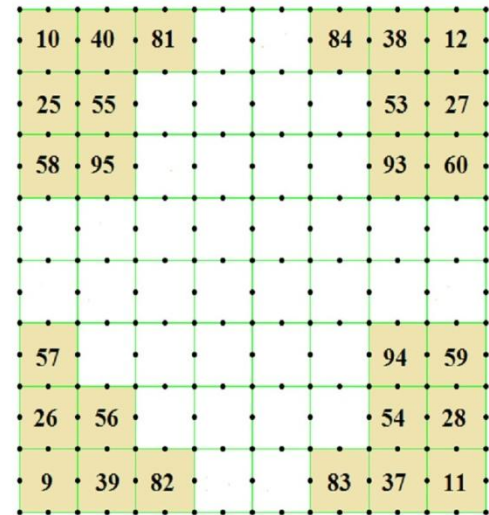


0° (FAILURE 4)

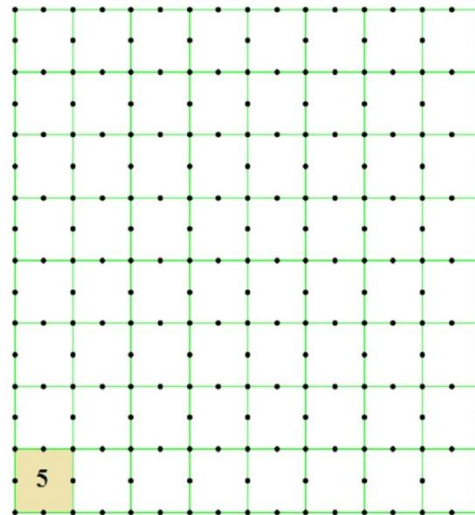
Fig. 7.16 Failure pattern of type: SYCP lamina for boundary condition: SSSS (Contd.)



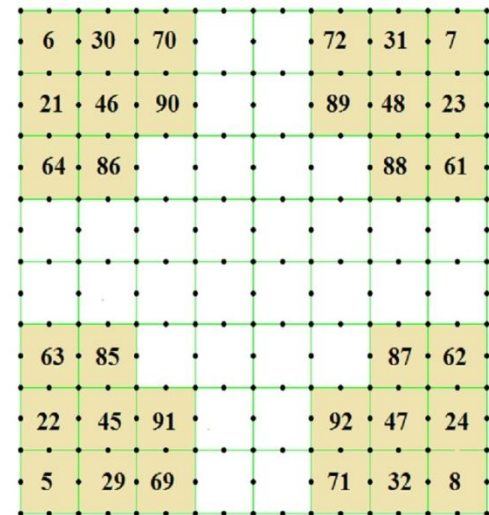
0° (FAILURE 5)



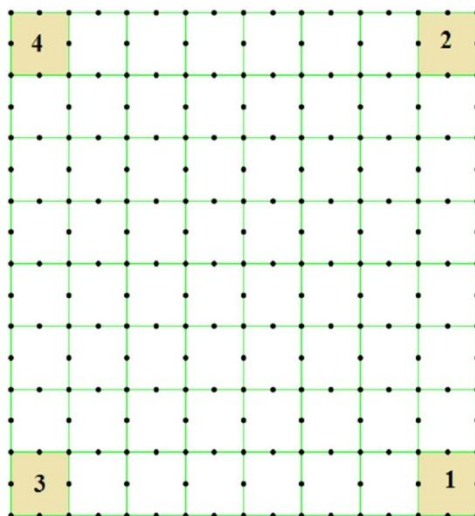
0° (0.25 UPF)



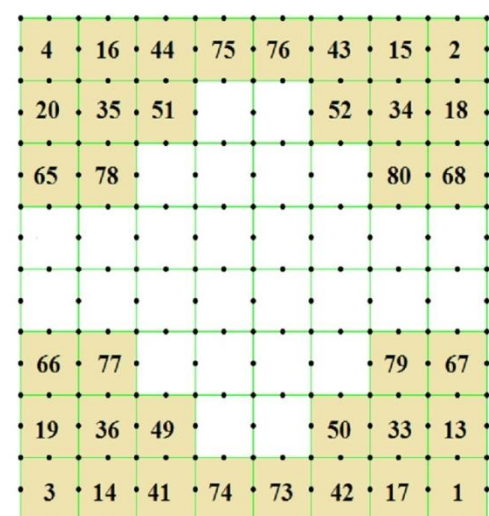
90° (FAILURE 5)



90° (0.25 UPF)

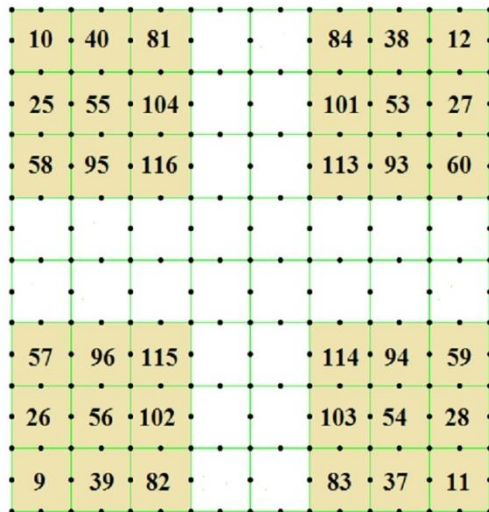


0° (FAILURE 5)

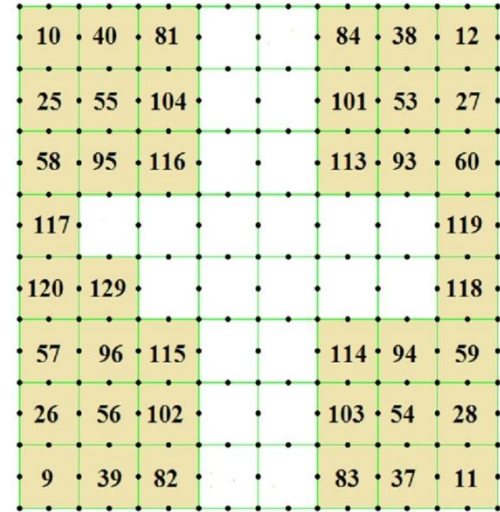


0° (0.25 UPF)

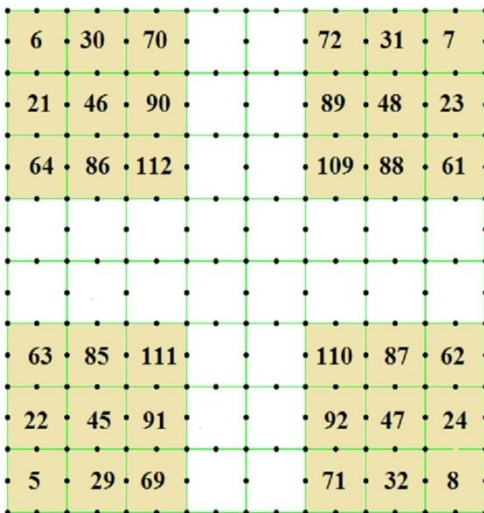
Fig. 7.16 Failure pattern of type: SYCP lamina for boundary condition: SSSS (Contd.)



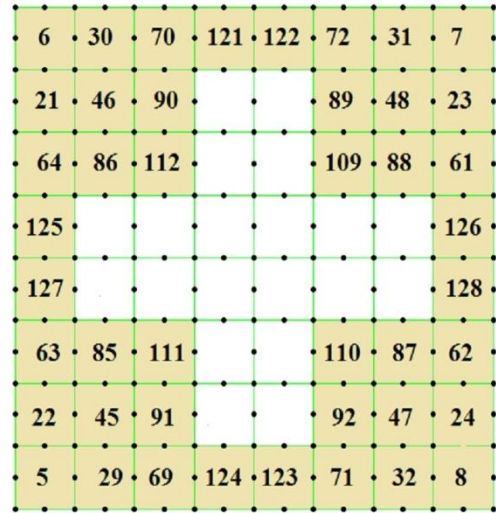
0° (0.5 UPF)



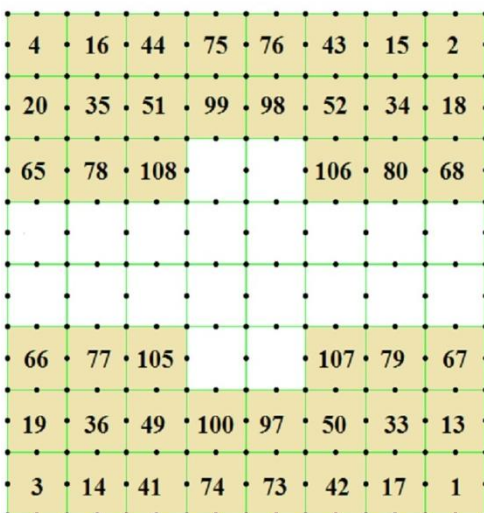
0° (0.75 UPF)



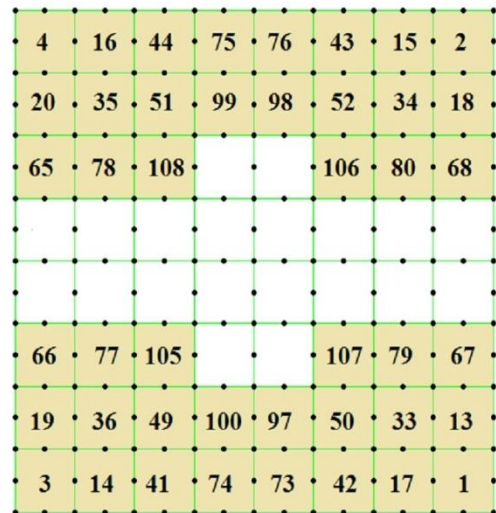
90° (0.5 UPF)



90° (0.75 UPF)



0° (0.5 UPF)



0° (0.75 UPF)

Fig. 7.16 Failure pattern of type: SYCP lamina for boundary condition: SSSS (Contd.)

10	40	81	138	139	84	38	12
25	55	104			101	53	27
58	95	116			113	93	60
117	132					130	119
120	129					131	118
57	96	115			114	94	59
26	56	102			103	54	28
9	39	82	137	140	83	37	11

0° (UPF)

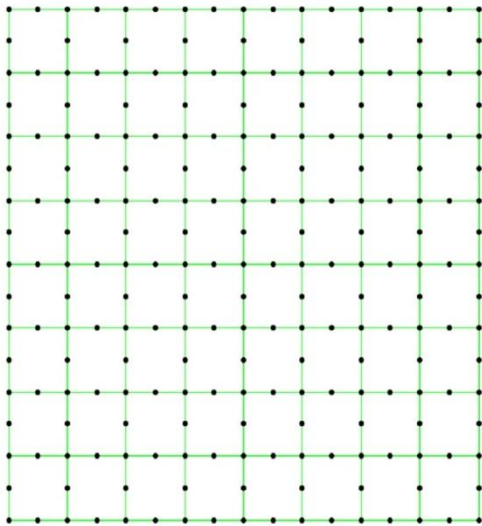
6	30	70	121	122	72	31	7
21	46	90	148	145	89	48	23
64	86	112			109	88	61
125	141					142	126
127	144					143	128
63	85	111			110	87	62
22	45	91	147	146	92	47	24
5	29	69	124	123	71	32	8

90° (UPF)

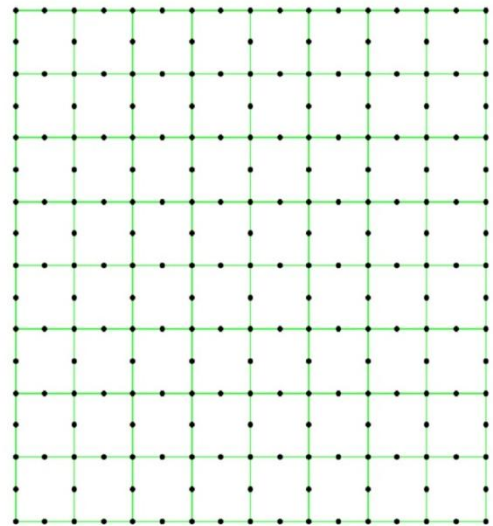
4	16	44	75	76	43	15	2
20	35	51	99	98	52	34	18
65	78	108			106	80	68
133							135
134							136
66	77	105			107	79	67
19	36	49	100	97	50	33	13
3	14	41	74	73	42	17	1

0° (UPF)

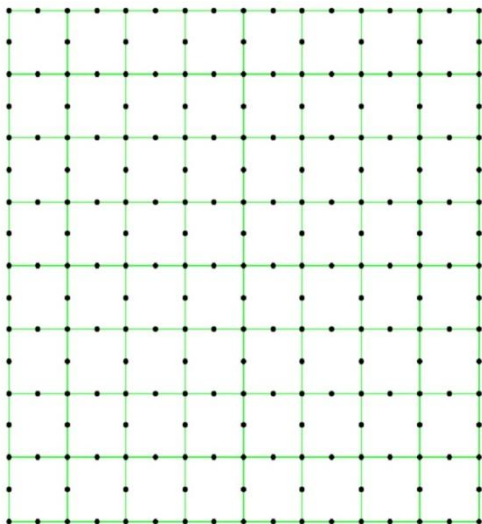
Fig. 7.17 Failure pattern of type: SYAP lamina for boundary condition: SSSS



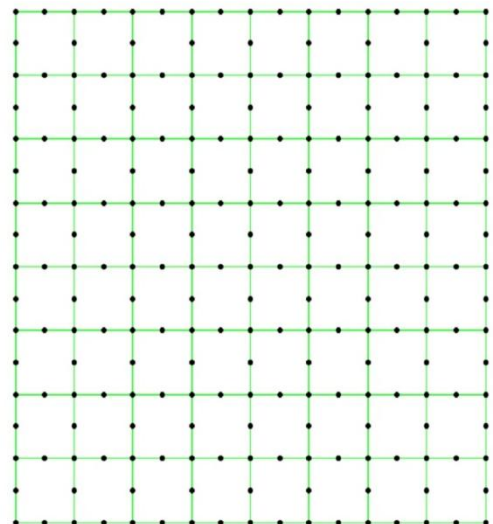
45° (FAILURE 1)



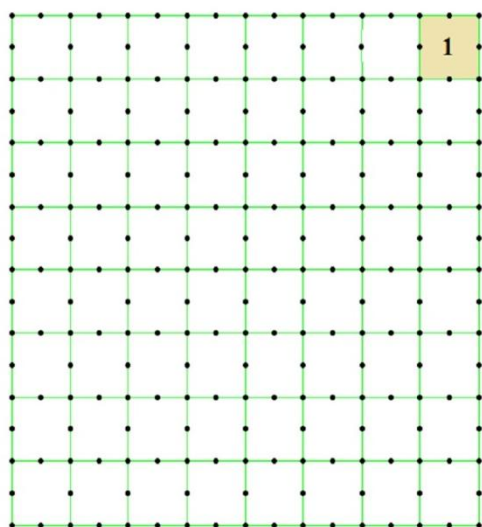
45° (FAILURE 2)



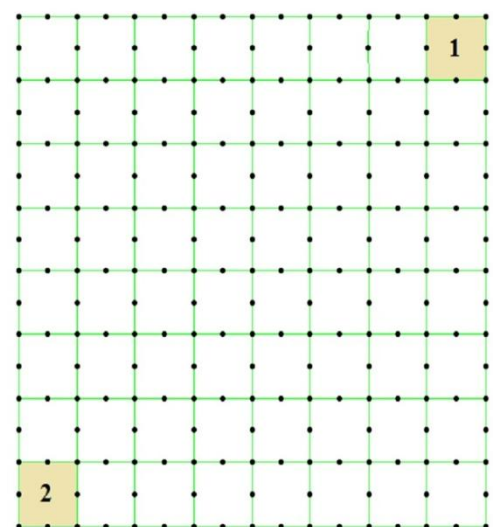
- 45° (FAILURE 1)



- 45° (FAILURE 2)

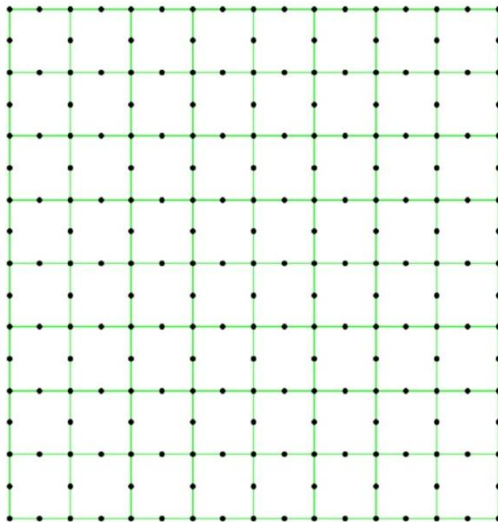


45° (FAILURE 1)

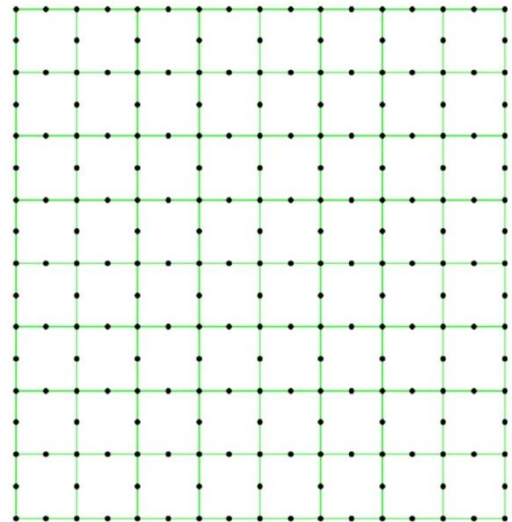


45° (FAILURE 2)

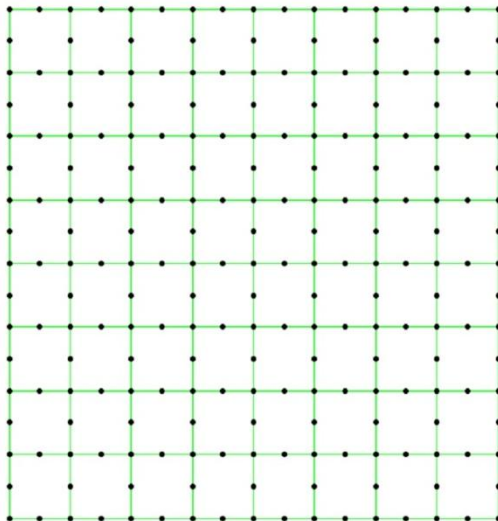
Fig. 7.17 Failure pattern of type: SYAP lamina for boundary condition: SSSS (Contd.)



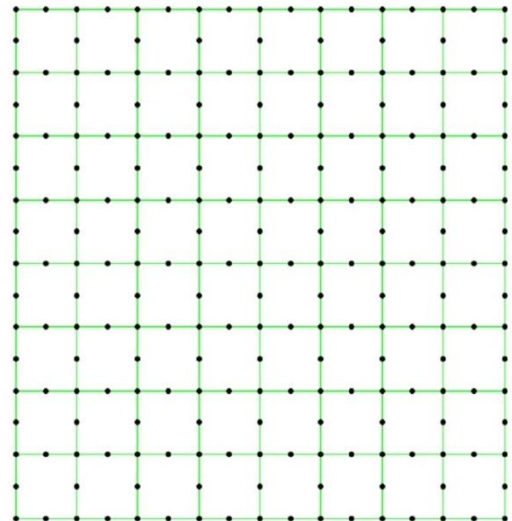
45° (FAILURE 3)



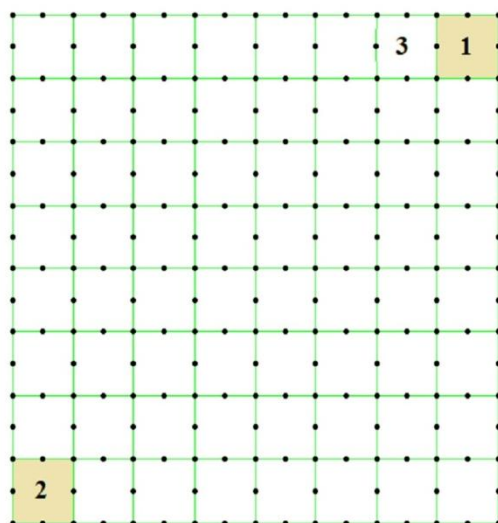
45° (FAILURE 4)



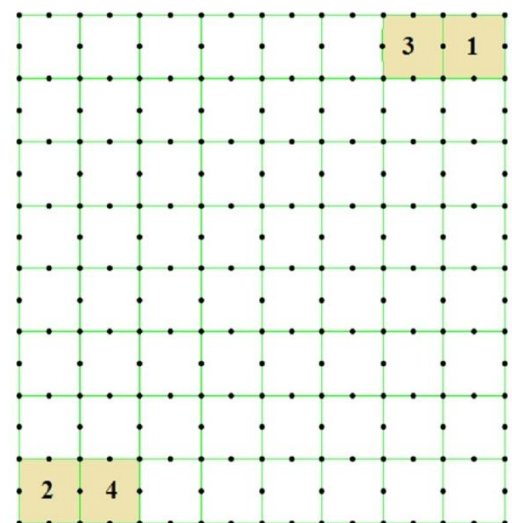
- 45° (FAILURE 3)



- 45° (FAILURE 4)

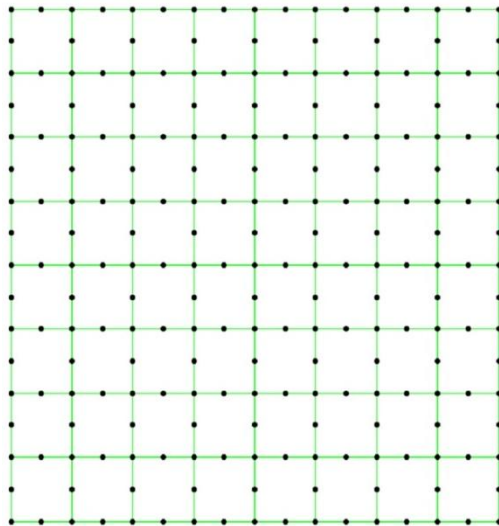


45° (FAILURE 3)

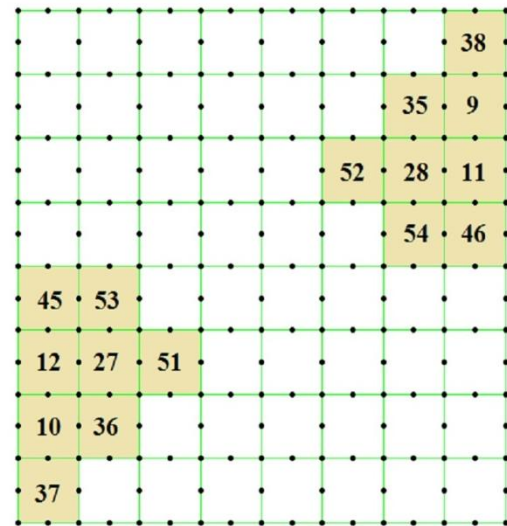


45° (FAILURE 4)

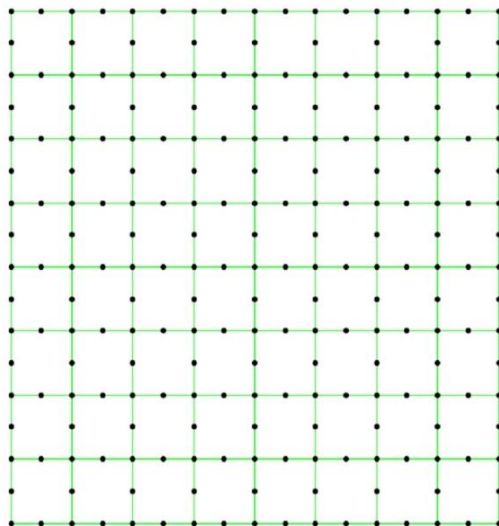
Fig. 7.17 Failure pattern of type: SYAP lamina for boundary condition: SSSS (Contd.)



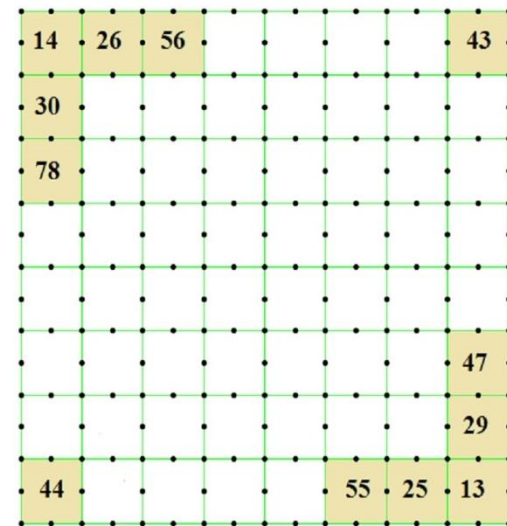
45° (FAILURE 5)



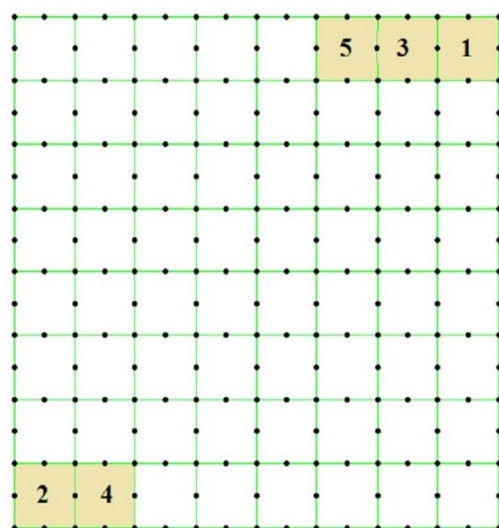
45° (0.25 UPF)



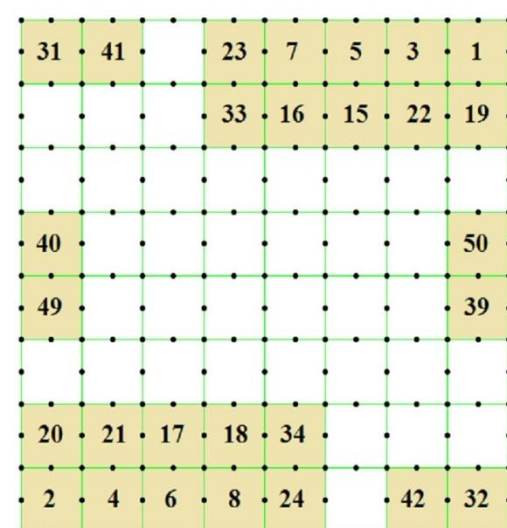
-45° (FAILURE 5)



-45° (0.25 UPF)

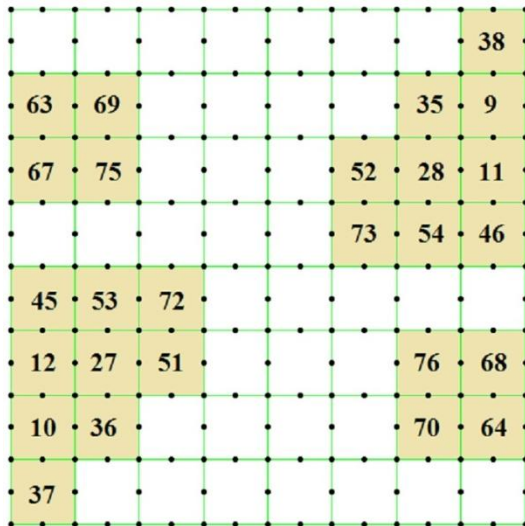


45° (FAILURE 5)

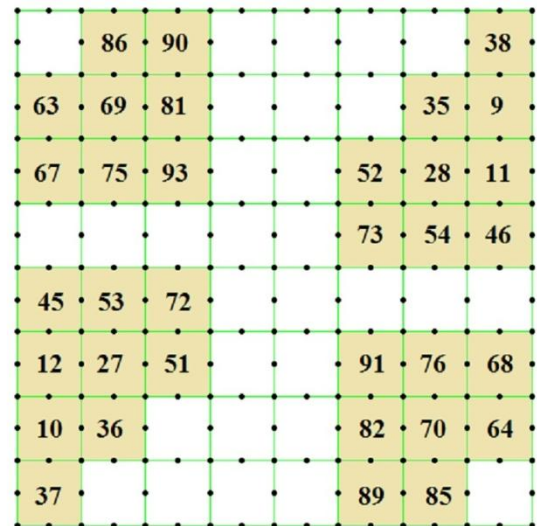


45° (0.25 UPF)

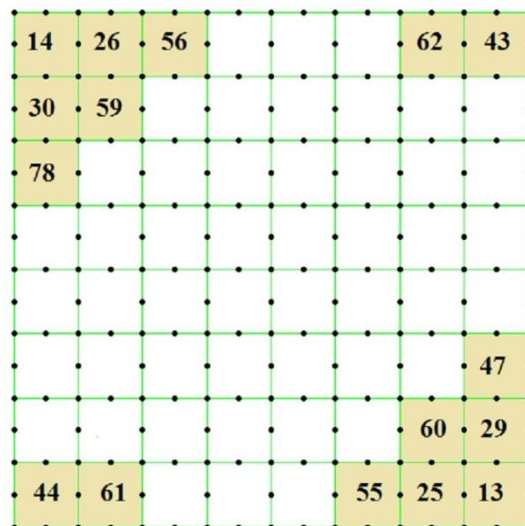
Fig. 7.17 Failure pattern of type: SYAP lamina for boundary condition: SSSS (Contd.)



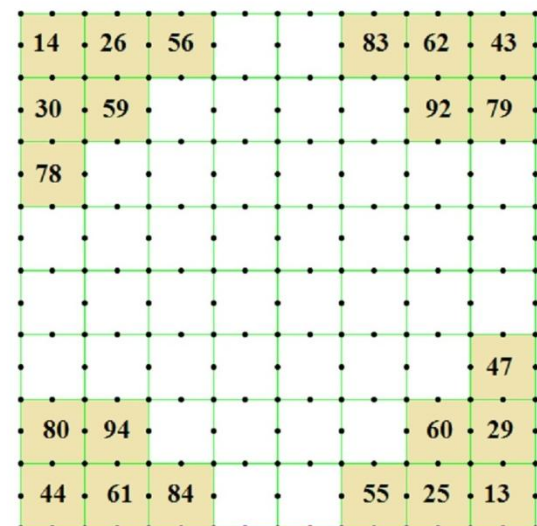
45° (0.5 UPF)



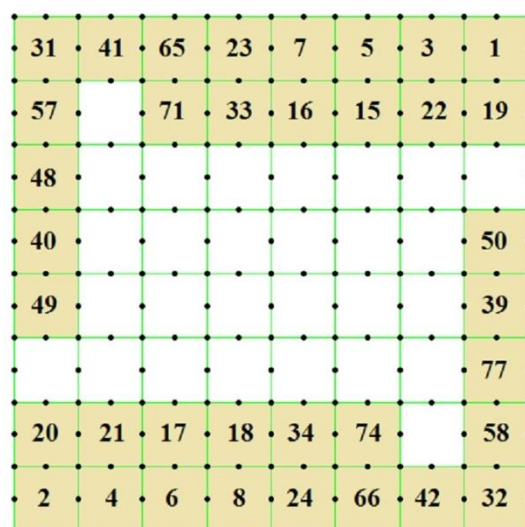
45° (0.75 UPF)



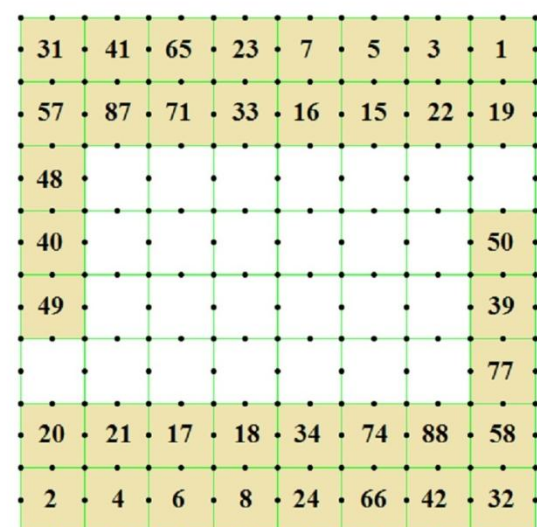
-45° (0.5 UPF)



-45° (0.75 UPF)

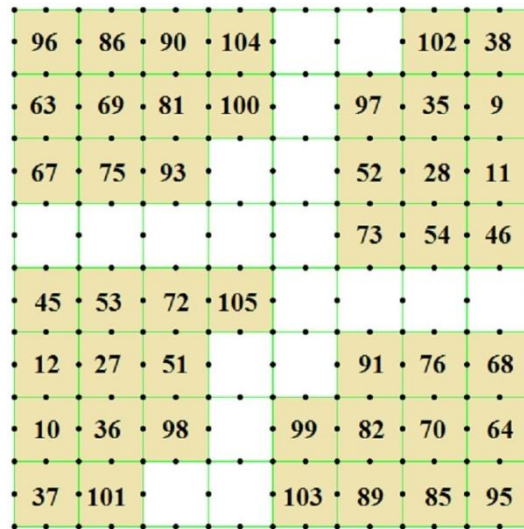


45° (0.5 UPF)

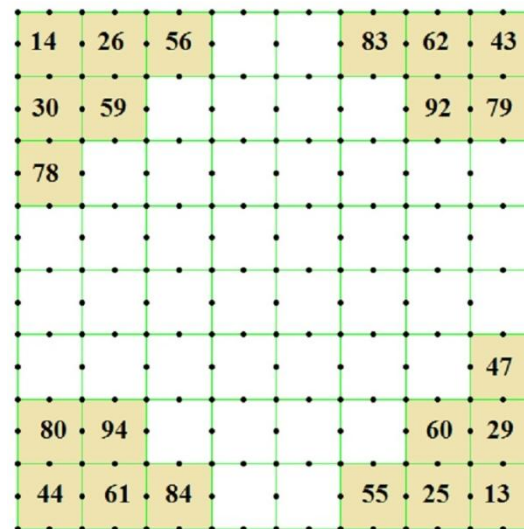


45° (0.75 UPF)

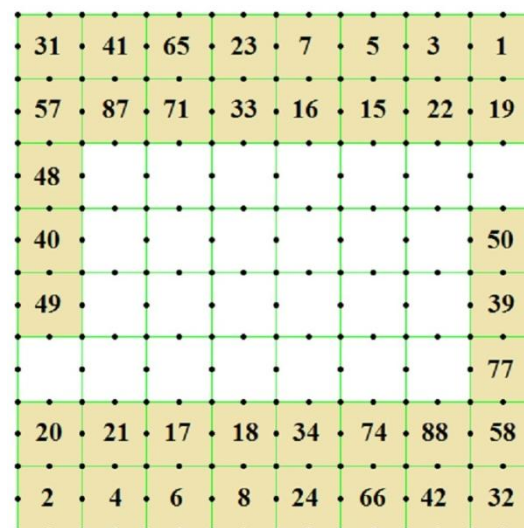
Fig. 7.17 Failure pattern of type: SYAP lamina for boundary condition: SSSS (Contd.)



45° (UPF)



-45° (UPF)



45° (UPF)

7.6 Correlation between Deflection and Area of Damage

Fig. 7.18 to 7.21 represents the variations of the cylindrical shells with the area of damage for all four laminations and boundary conditions. It is found that the curves are simple ones either linear or parabolic and working equations can be established between the central deflection and area of damage. Since measurements of deflection is easy by setting up simple dial gauges these equations furnished in the Table 7.11 are expected to be very useful to practicing engineers to predict the extends of damages from a knowledge of measured deflections.

Table 7.11 Explicit equations between central deflections (C) and percentage of total damaged area (A)

Boundary Condition	Lamination	Relation	Equations correlating central deflections in mm (C) and percentage of total damaged area (A)	R ²
CCCC	ASCP	Parabolic	$C = 0.0047A^2 - 0.5156A - 4.2008$	0.9592
	ASAP	Linear	$C = -0.1803A - 1.8665$	0.9549
	SYCP	Parabolic	$C = 0.0367A^2 - 1.1904A - 6.826$	0.8774
	SYAP	Parabolic	$C = 0.0155A^2 - 0.7628A - 3.3559$	0.9476
CSSC	ASCP	Parabolic	$C = -0.0045A^2 - 0.0902A - 0.1993$	0.9722
	ASAP	Linear	$C = 0.4229A + 0.7809$	0.9869
	SYCP	Parabolic	$C = -0.0296A^2 + 0.3957A - 5.7073$	0.9881
	SYAP	Linear	$C = 0.3899A + 0.4599$	0.9918
SCSC	ASCP	Parabolic	$C = 0.011A^2 - 0.6527A - 3.6862$	0.9194
	ASAP	Linear	$C = 0.1553A + 2.7521$	0.9496
	SYCP	Parabolic	$C = 0.0222A^2 - 0.8356A - 6.5953$	0.8332
	SYAP	Parabolic	$C = -0.0012A^2 + 0.0996A + 0.5809$	0.9598
SSSS	ASCP	Linear	$C = -0.1315A + 0.8149$	0.8968
	ASAP	Linear	$C = 0.6262A - 0.8273$	0.9721
	SYCP	Parabolic	$C = -0.0103A^2 + 0.1381A - 9.2154$	0.9795
	SYAP	Linear	$C = 0.6504A + 0.9361$	0.9721

Fig. 7.18(a) Central displacement Vs. Area of damage curve for boundary condition: CCCC

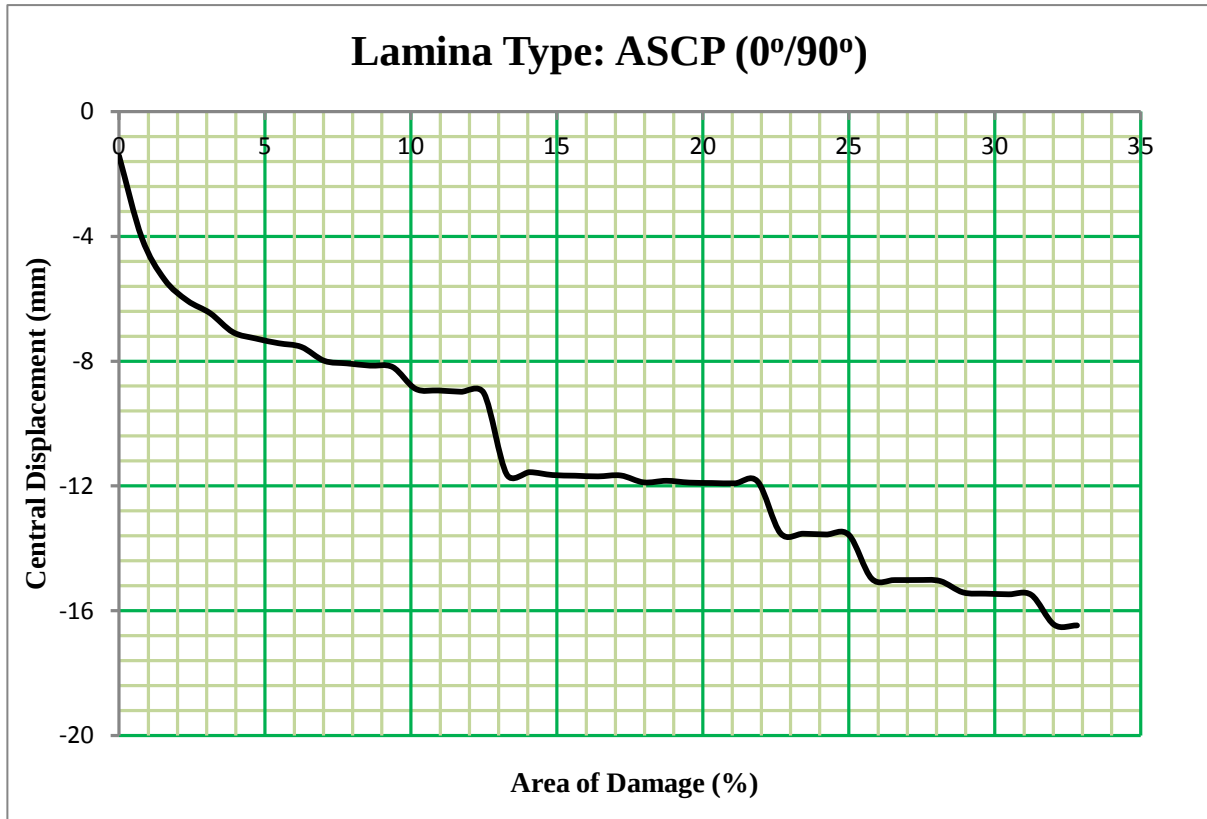


Fig. 7.18(b) Central displacement Vs. Area of damage curve for boundary condition: CCCC

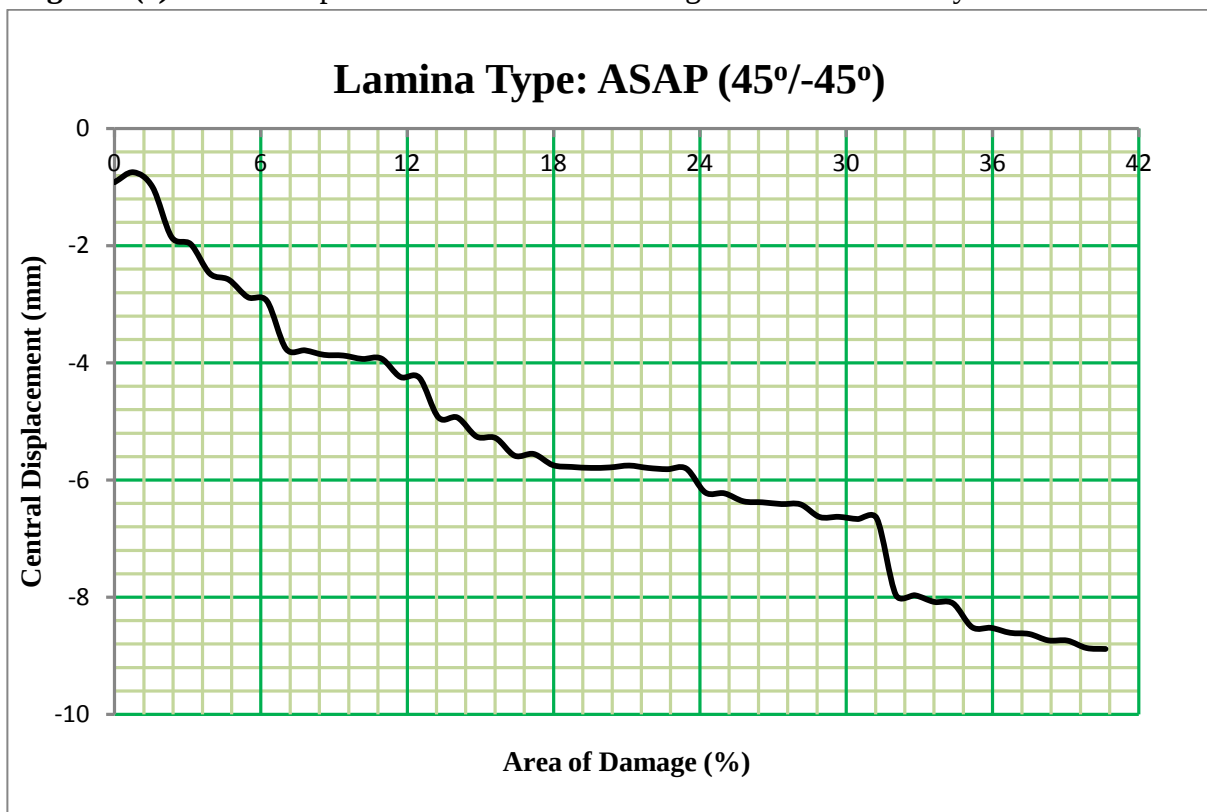


Fig. 7.18(c) Central displacement Vs. Area of damage curve for boundary condition: CCCC

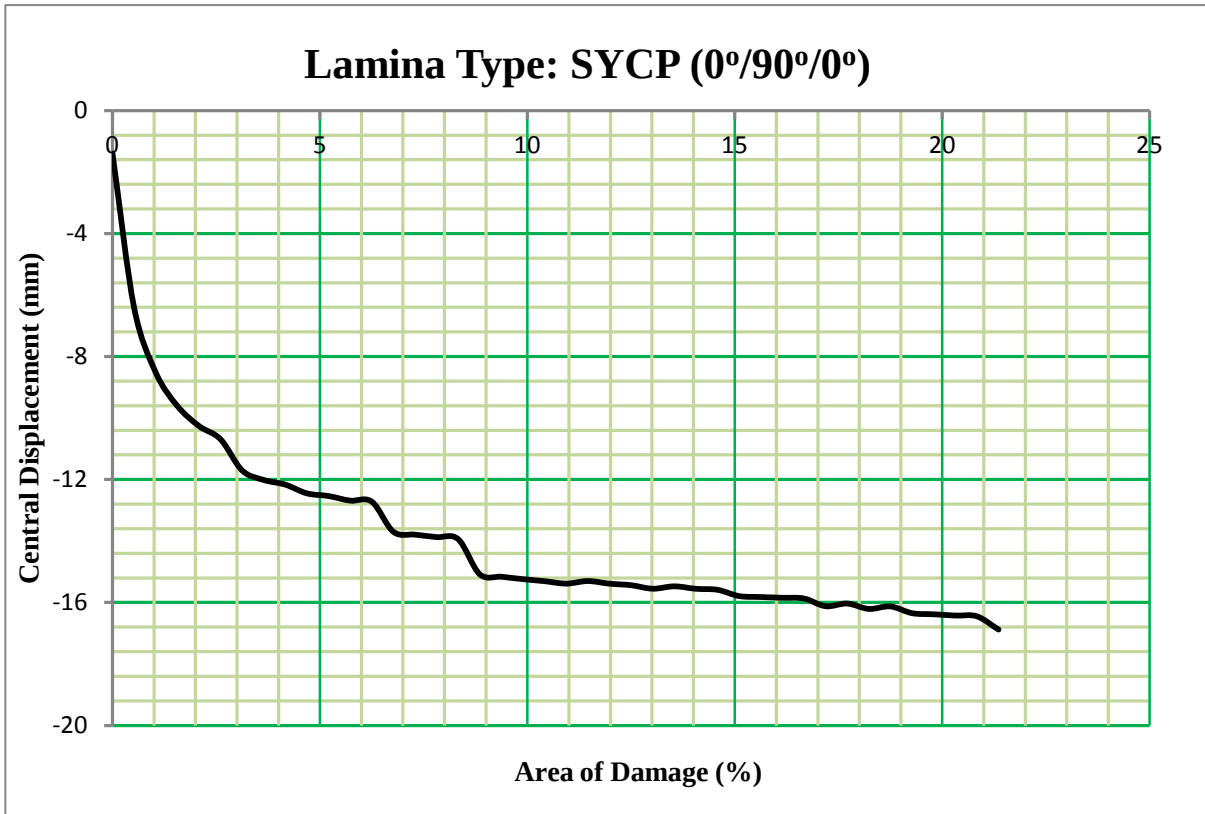


Fig. 7.18(d) Central displacement Vs. Area of damage curve for boundary condition: CCCC

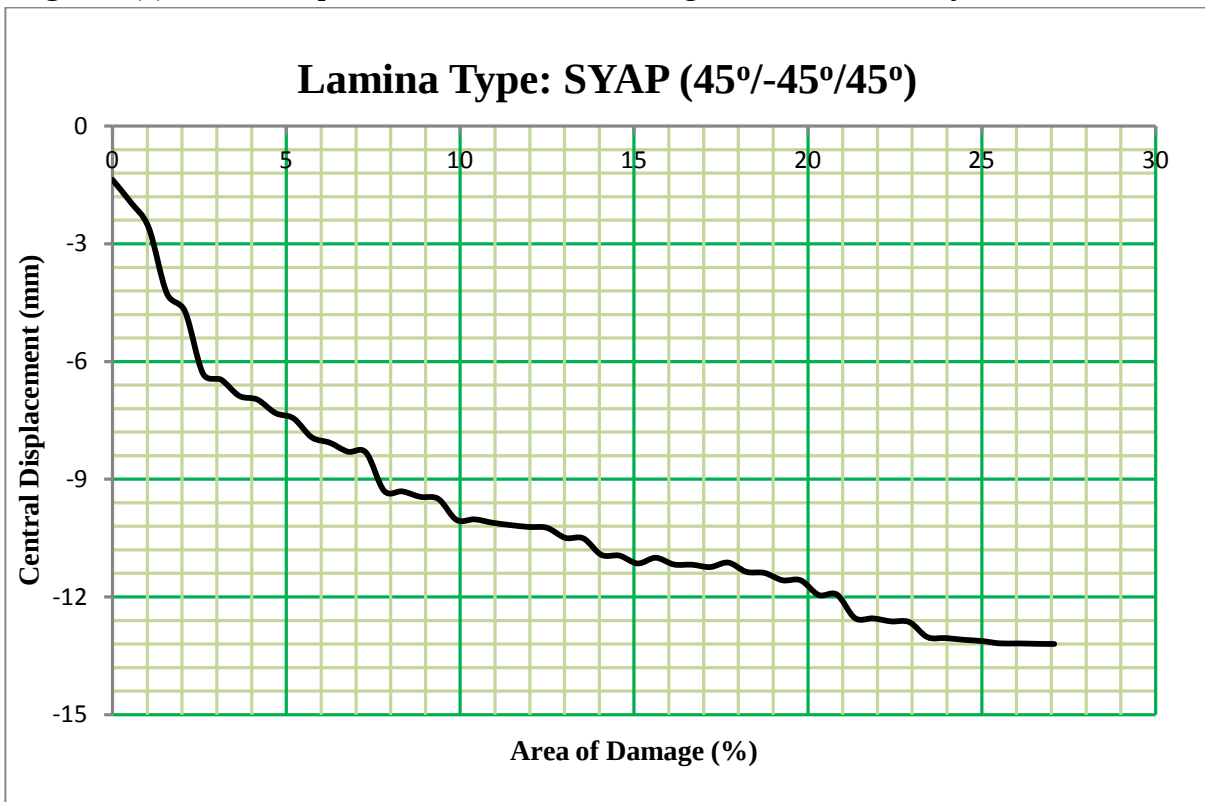


Fig. 7.19(a) Central displacement Vs. Area of damage curve for boundary condition: CSSC

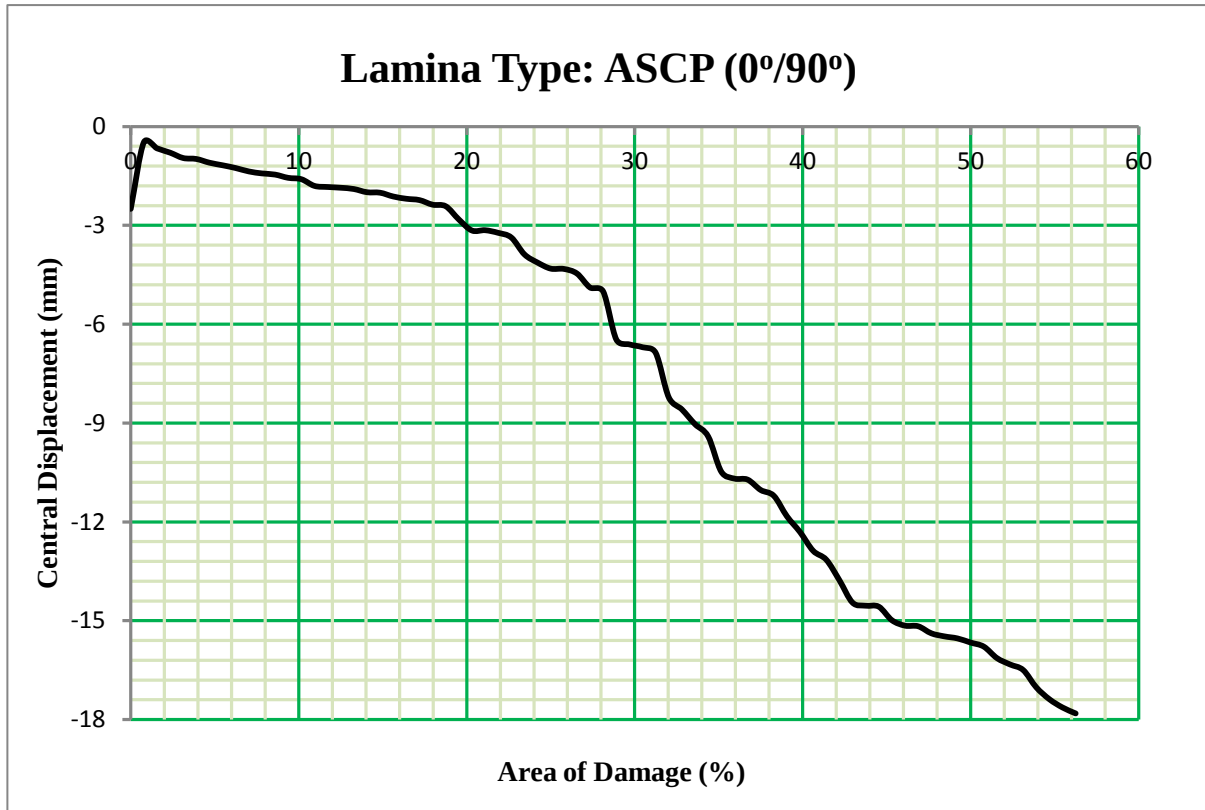


Fig. 7.19(b) Central displacement Vs. Area of damage curve for boundary condition: CSSC

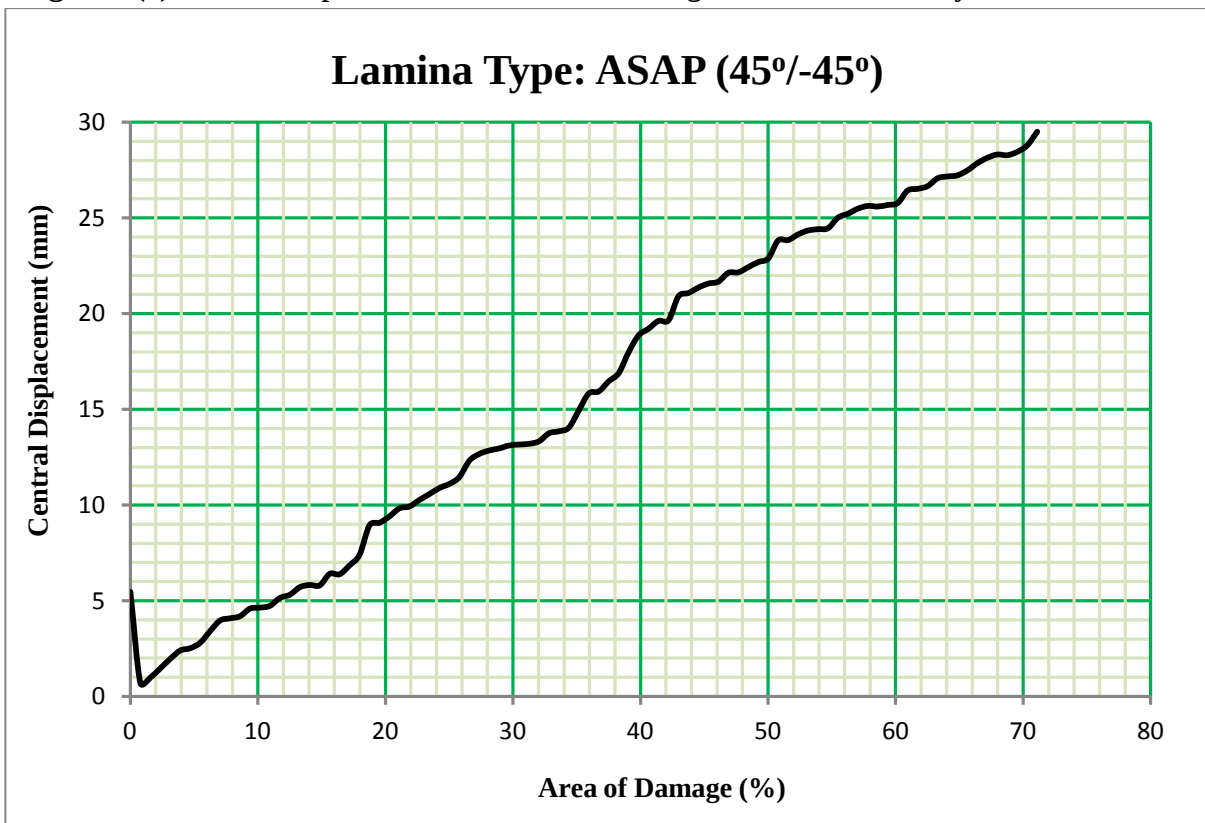


Fig. 7.19(c) Central displacement Vs. Area of damage curve for boundary condition: CSSC

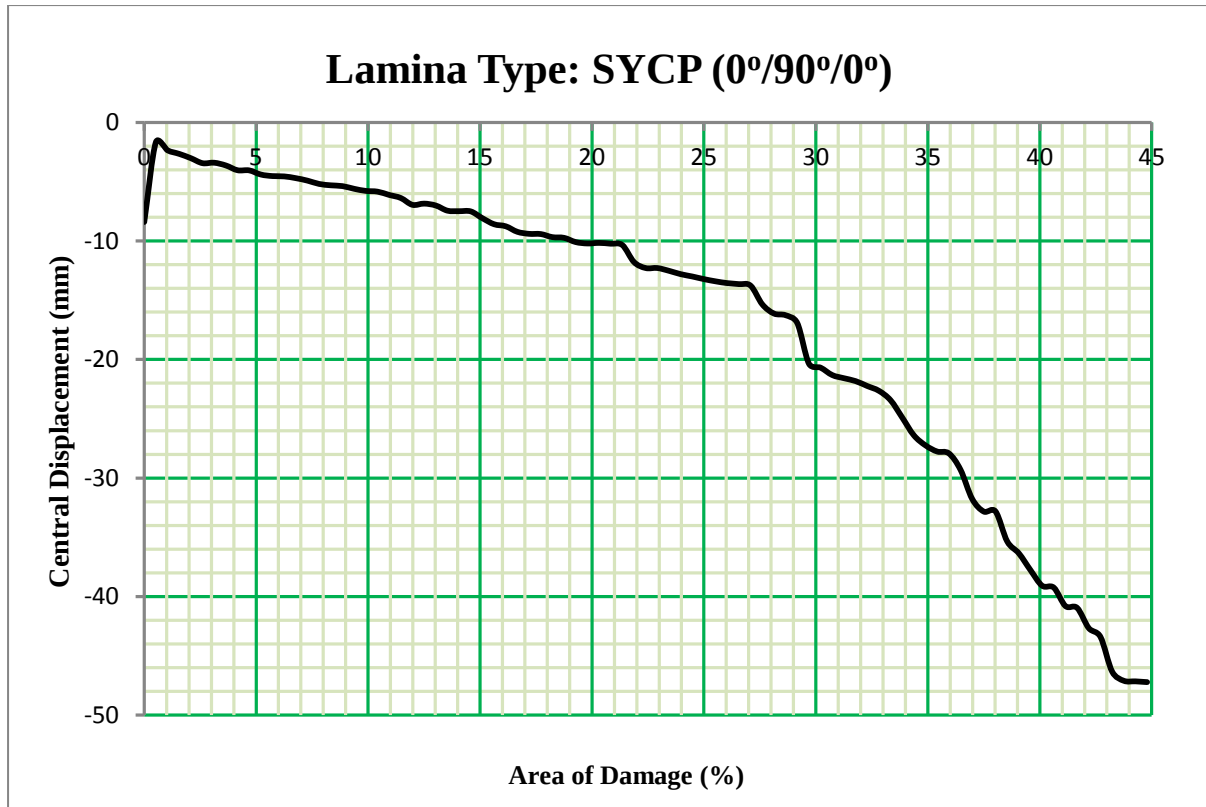


Fig. 7.19(d) Central displacement Vs. Area of damage curve for boundary condition: CSSC

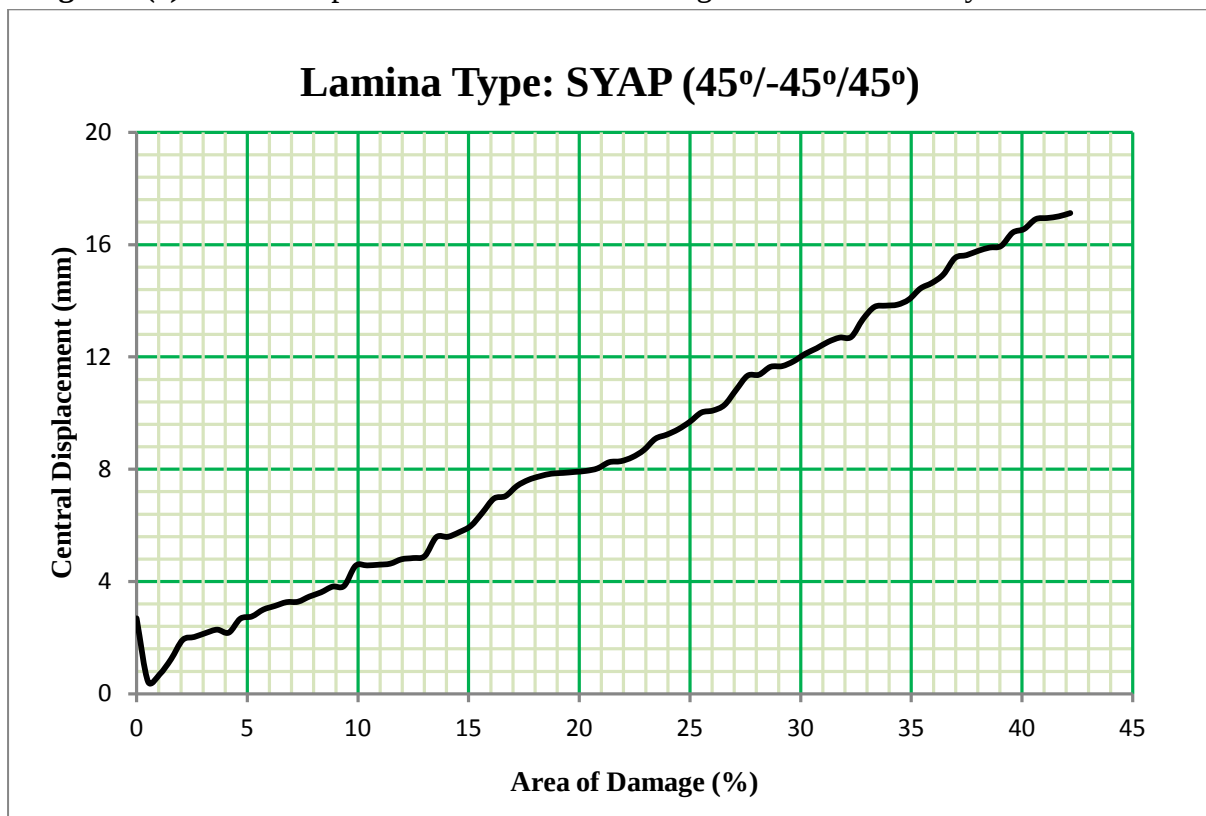


Fig. 7.20(a) Central displacement Vs. Area of damage curve for boundary condition: SCSC

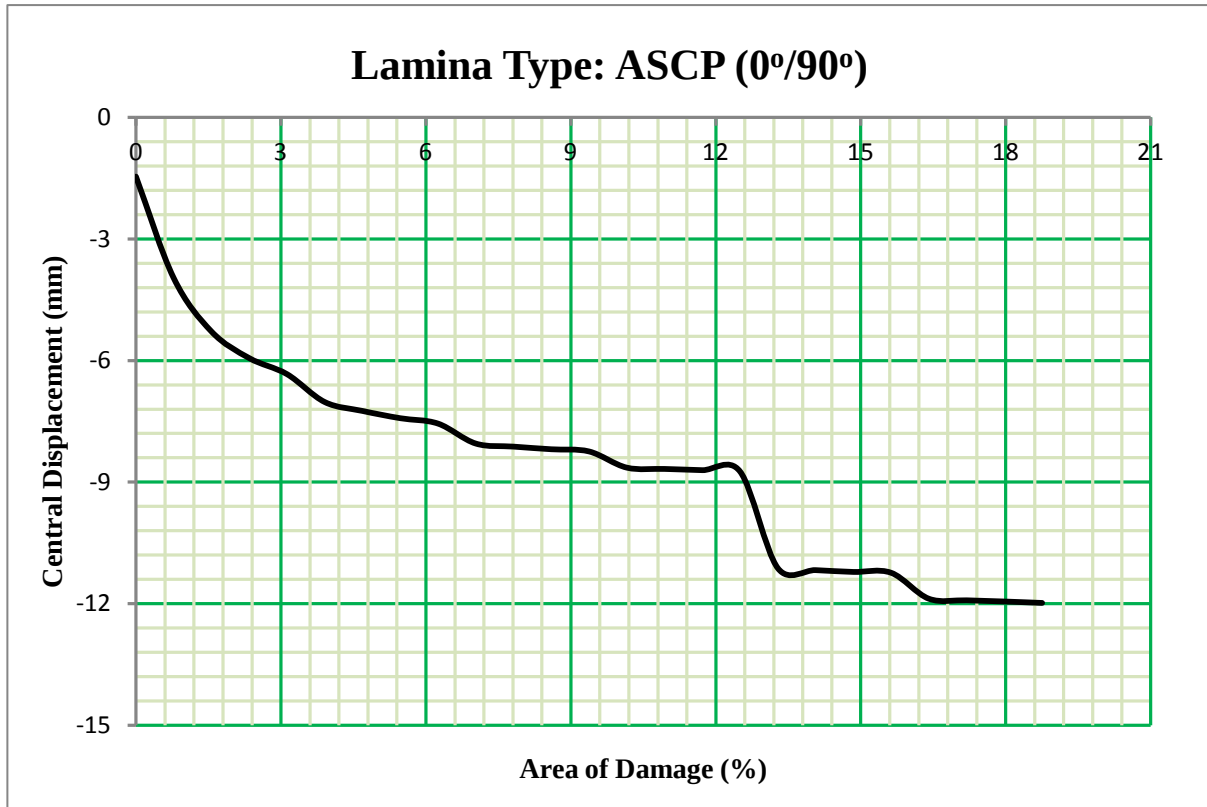


Fig. 7.20(b) Central displacement Vs. Area of damage curve for boundary condition: SCSC

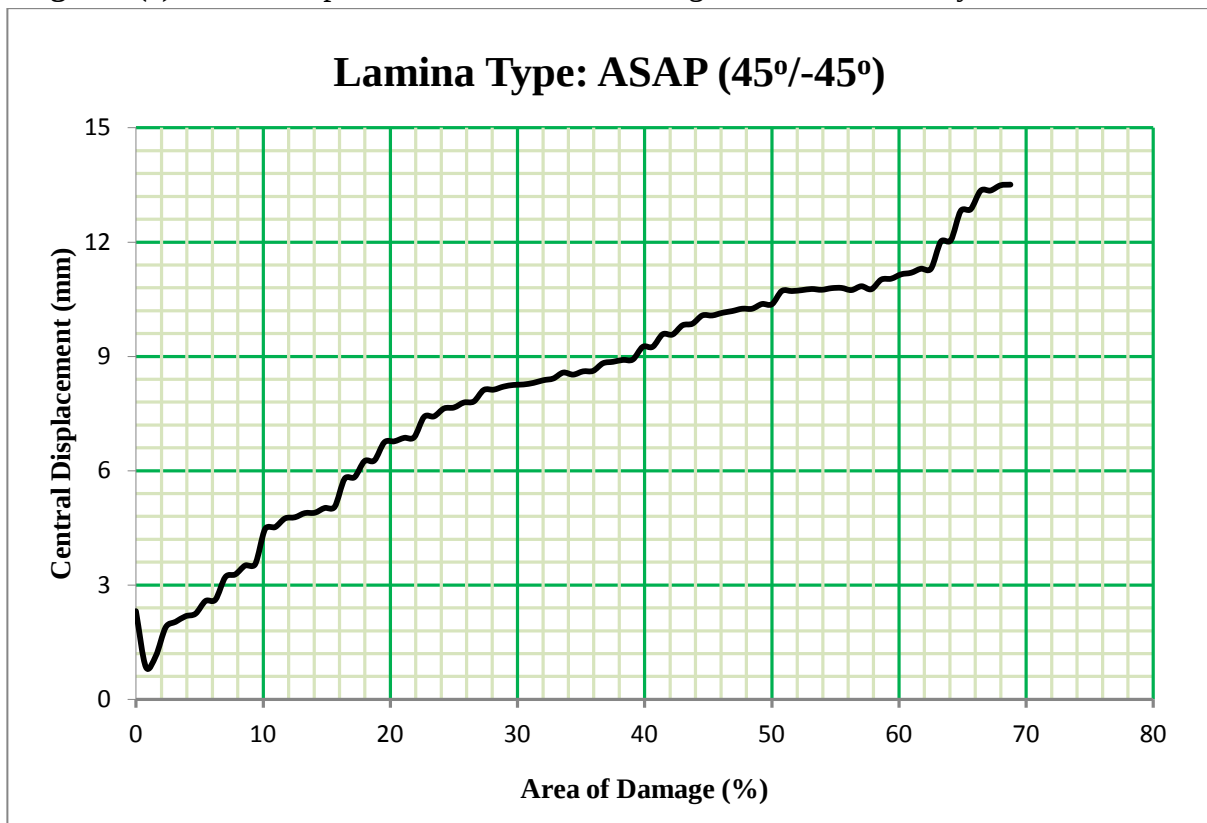


Fig. 7.20(c) Central displacement Vs. Area of damage curve for boundary condition: SCSC

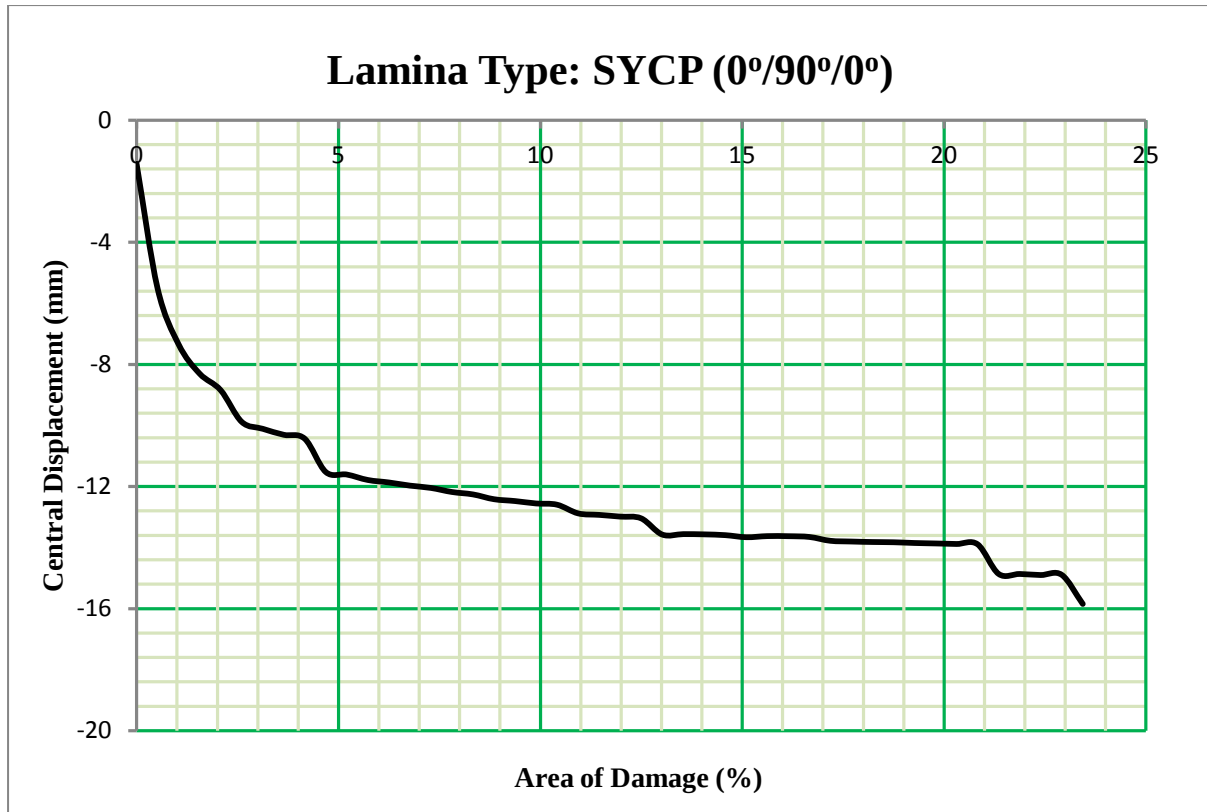


Fig. 7.20(d) Central displacement Vs. Area of damage curve for boundary condition: SCSC

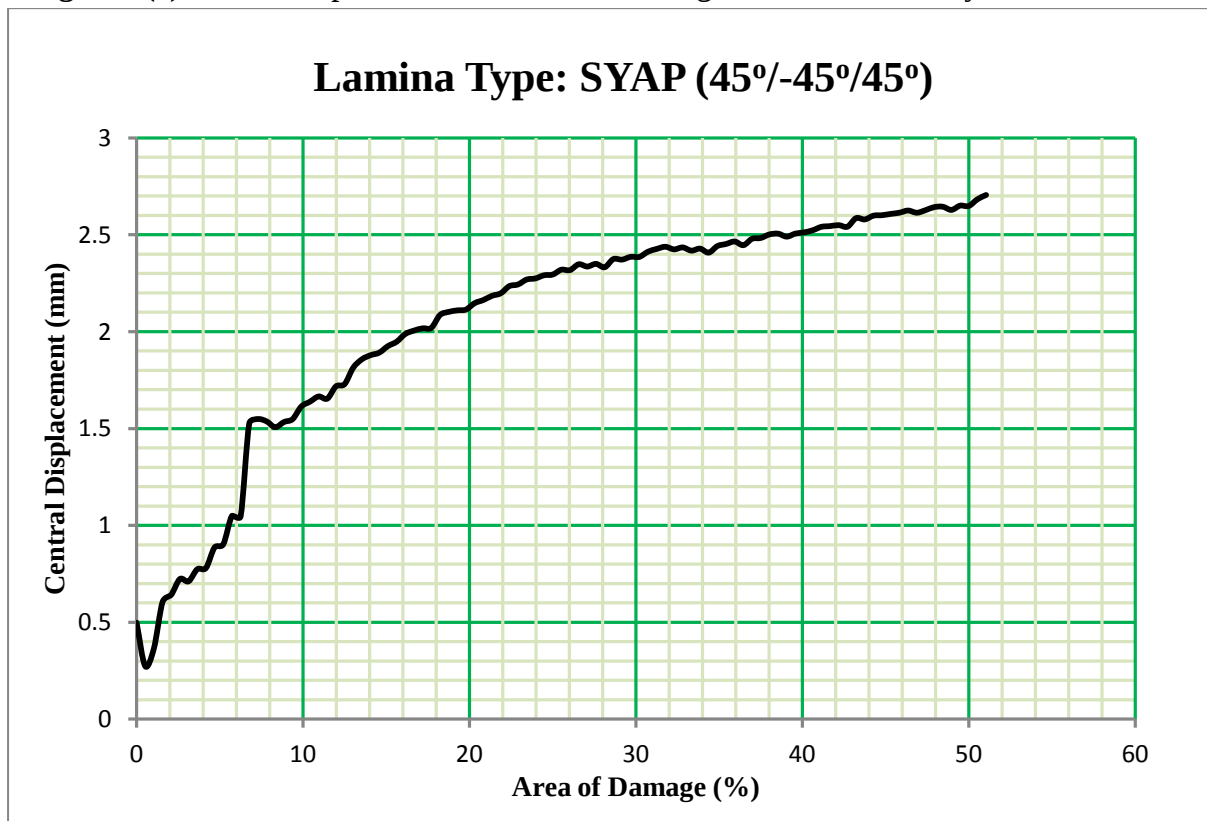


Fig. 7.21(a) Central displacement Vs. Area of damage curve for boundary condition: SSSS

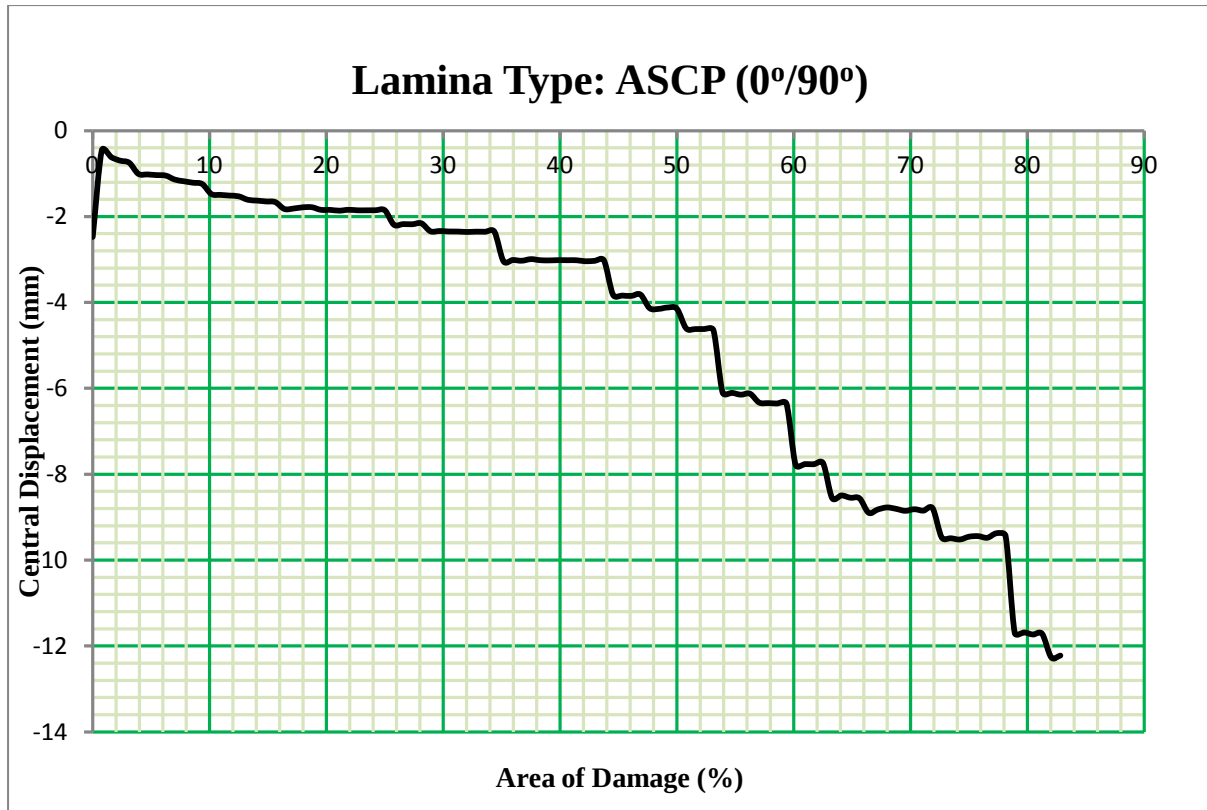


Fig. 7.21(b) Central displacement Vs. Area of damage curve for boundary condition: SSSS

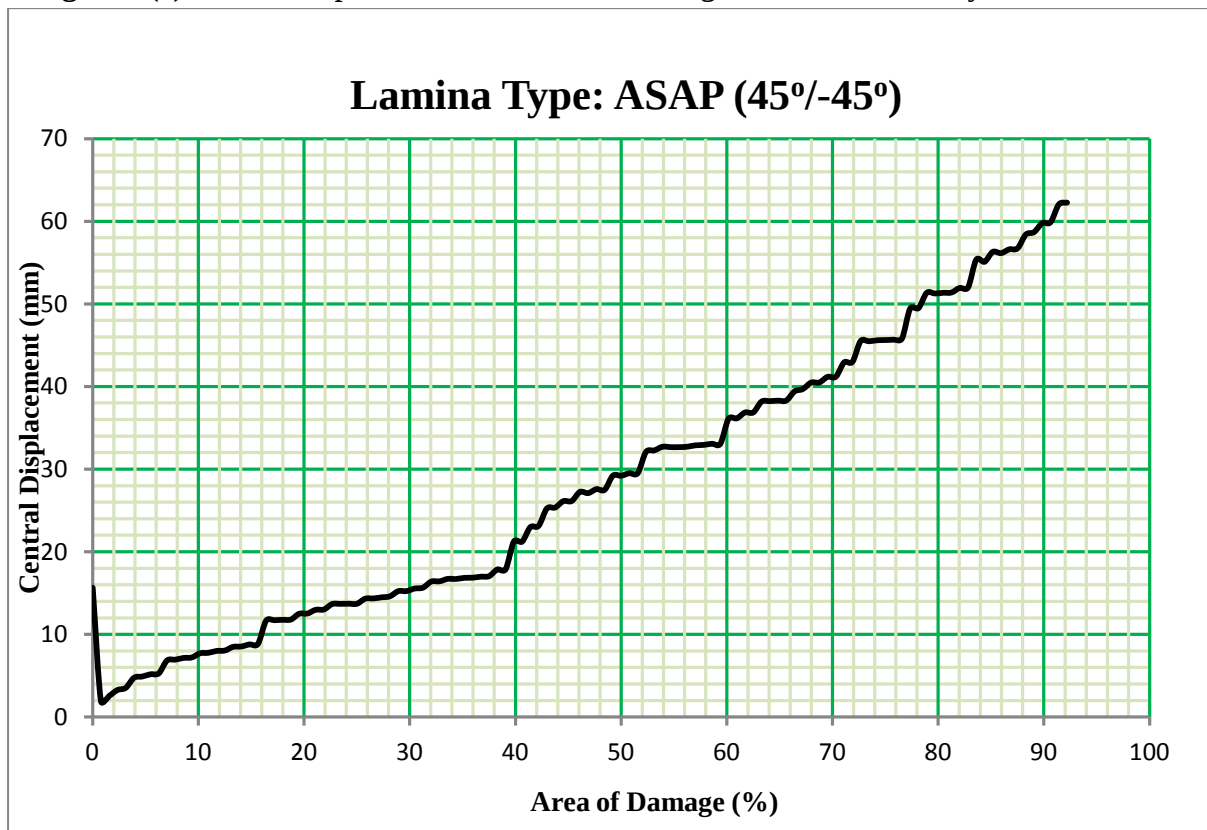


Fig. 7.21(c) Central displacement Vs. Area of damage curve for boundary condition: SSSS

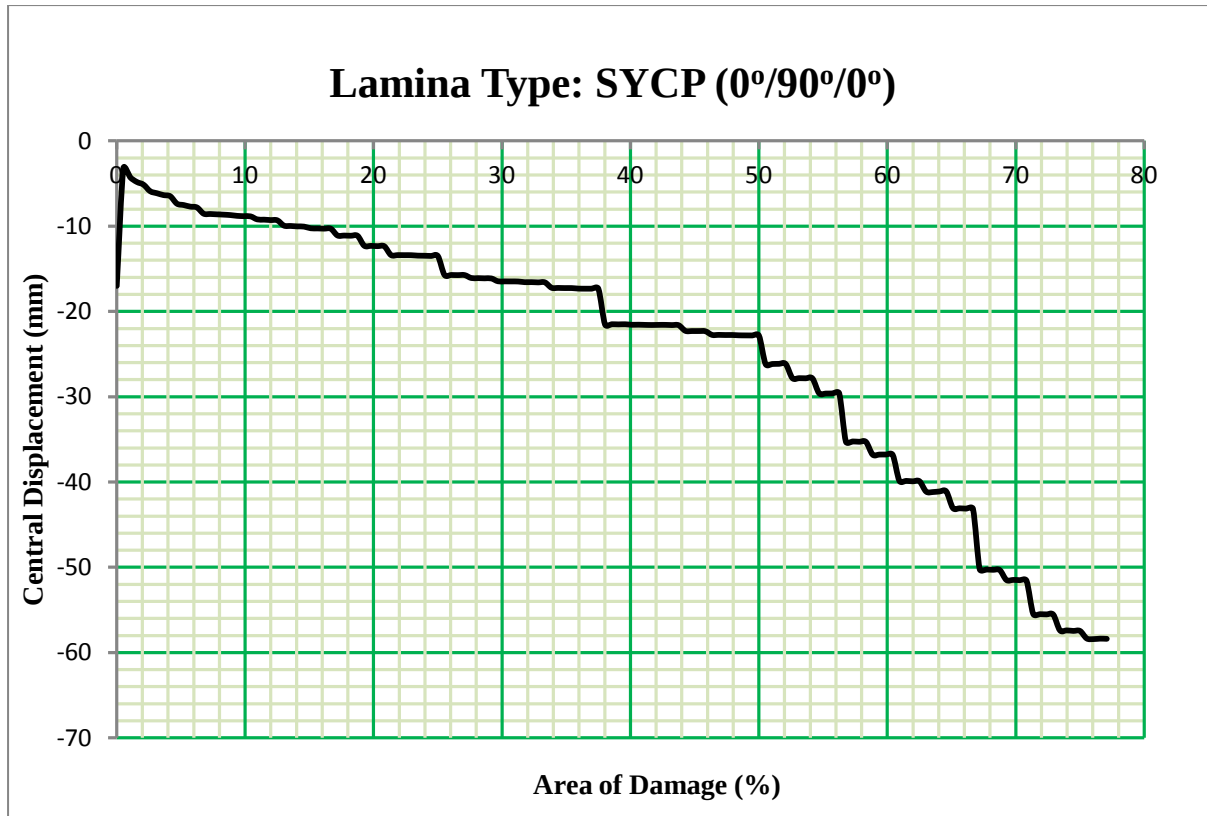
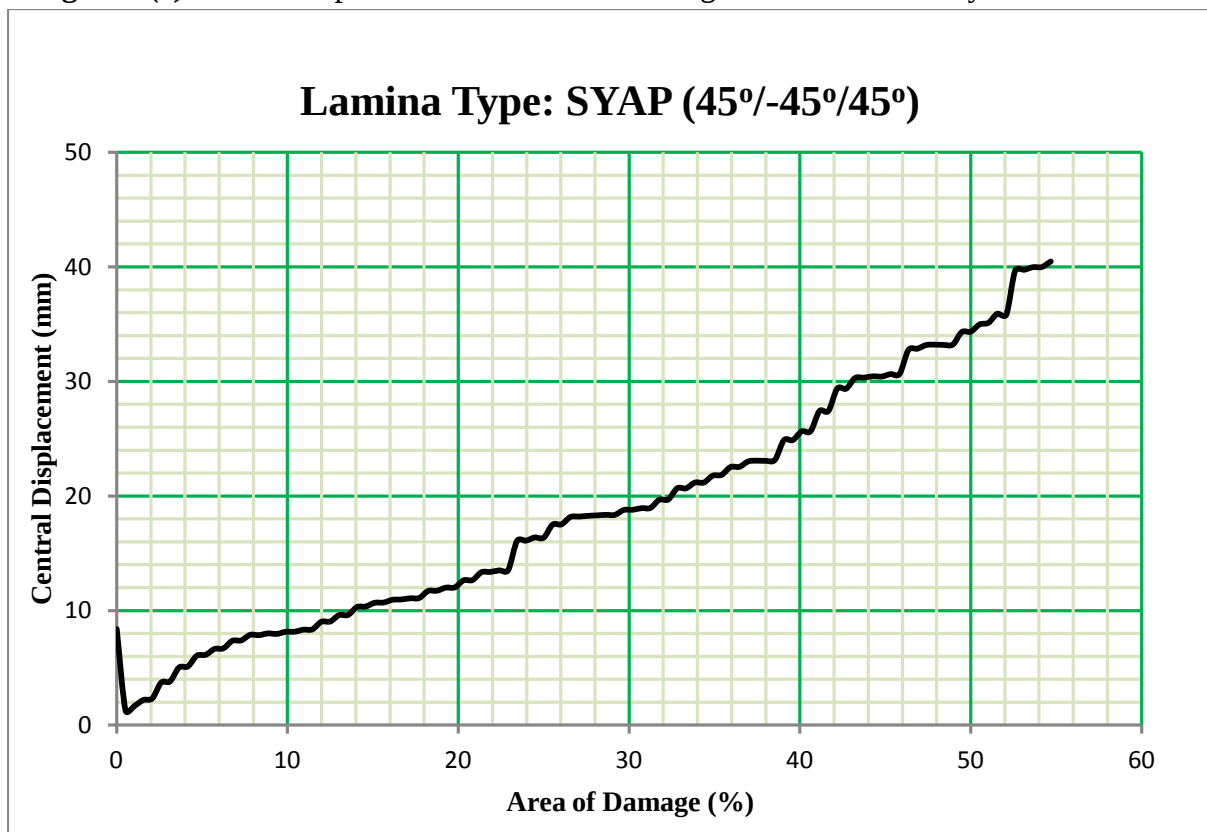


Fig. 7.21(d) Central displacement Vs. Area of damage curve for boundary condition: SSSS



7.7 Flaw Detection with the Understanding of Failure Propagation

Figures 7.2 to 7.17 depict the lamina-wise failure progress mechanism with gradually increasing failure load. This knowledge may further be used for getting a warning in advance about the gross failure of the shell by appropriate instrumentation related to different non destructive tests (NDT). Some of the NDT methods generally applied to laminated composites are the Pulse Echo method, Through Transmission method, Back Scattering method, Acousto-Ultrasonic and Ultrasonic Spectroscopy methods etc. If an engineer has a prior knowledge regarding the flow of damage then appropriate points on the shell surfaces may be tested only to detect the hidden flaws without going into an elaborate testing programme. The maximum area of damage can be allowed under a practical situation may be correlated with the deflection value as discussed before and may also be correlated with the NDT results at predetermined points.

For example suppose, in case of a clamped $0^\circ/90^\circ$ shell, one has an ultrasonic flaw detection point set up at $x = 875$ mm, $y = 1000$ mm; once a flaw is detected at that point, one can be sure that area of damage is about 4.6875 %. In other words if he wants to restrict the shell within a maximum limit of 5% (say) damage he should be vigilant about the ultrasonic flaw detection results only at the point as mentioned above without going into elaborate instrumentation.

Chapter 8

CONCLUSION

8.1 General

The first and ultimate ply failure loads of laminated composite cylindrical shell roofs are studied in the present thesis. Section 8.2 presents the major conclusions point wise and Section 8.3 indicates the future scope of the present work.

8.2 Conclusions

- a) The finite element code developed and applied here, can be accepted as a successful tool to explore the first and ultimate ply failure aspects of composite cylindrical shells. Solutions obtained for the benchmark problems using the present method indicate this fact.
- b) In general the performances of the cross ply shells perform better than the angle ply ones and the symmetric laminates are better than their anti-symmetric counterparts in terms of first ply failure loads. However the angle ply shells offer better performance for ultimate ply failure requisites.
- c) It may be inferred from the results that among two laminates the one stiffer in terms of FPF (first ply failure) may not be the stiffer one in terms of the UPF (ultimate ply failure) load.
- d) When the failure loads corresponding to all the boundary conditions and laminations taken up are compared, it is found that the fully clamped shells yield the highest value of FPFL while the simply supported ones exhibit the lowest values. This is quite expected because the number of boundary constraint determines the overall rigidity of the structure and in that sense the clamped shell has the maximum degrees of freedom locked and hence is the stiffest choice.
- e) The SCSC shells have the two beam edges clamped and have much more improved failure load values compared to the CSSC ones. This is because a cylindrical shell transfers the loads and moments along two plan directions parallel to its principal curvatures and when the fibres too are oriented in these directions as in cross ply shells, the overall improvement of its behaviour is noteworthy.
- f) It is observed that the values of LF (load factor) are extremely high and indicate that the laminated composites have substantial amount of load resisting capacity beyond the

working load and hence these materials are expected to exhibit a ductile failure making them appropriate to be used in earthquake prone zones.

- g) The values of PSF are different for different boundary conditions. Although for each of the boundary conditions such factors for different laminations have close values. If a single value of PSF (partial safety factor) is to be proposed then it should be adopted as 4.
- h) The general trend which is observed is that the deflection varies nonlinearly with the area of damage. So, explicit equations may be suggested correlating the deflections with area of damages when the damage is between certain limits.
- i) Knowledge of failure propagation may be used for getting a warning in advance about the gross failure of the shell by appropriate instrumentation related to different non – destructive tests (NDT). If an engineer has a prior knowledge regarding the flow of damage then appropriate points on the shell surfaces may be tested only to detect the hidden flaws without going into an elaborate testing programme. The maximum area of damage can be allowed under a practical situation may be correlated with the deflection value as discussed before and may also be correlated with the NDT results at predetermined points.

8.3 Future Scope

The critical discussion of the review of the accumulated literature, mentioned in Chapter 3, highlights the different areas which needs attention, from shell researches. In this section, the areas to which, the present work can directly be extrapolated, are mentioned. For example, point supported boundary cylindrical shell may be taken up for failure study. Shell surfaces are often stiffened for altering the resonating frequency and at the point of concentration loads. Hence failure studies of cylindrical shells under point loads need to be explored. In the industry, one often finds, shell surfaces with cut-outs for allowing, entry of light, air and passage of cables, conduits and chimney shafts. The present study may be extended to explore failure of shells with cutouts. Study of such shells for failure defines yet another area where the present work may be extended in future. There are many other areas which can be touched either by direct extension of present work or by minor alteration and modification of it. Nonlinear failure study may be carried out for each of the above mentioned cases.

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